

THE ACCURATE ULTRASONIC MEASUREMENT OF THE VOLUME FLOW
OF BLOOD BY TIME DOMAIN CORRELATION

BY

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A new ultrasonic flow measurement method employing time domain correlation of consecutive pairs of echoes has been developed. An ultrasonic data acquisition system determines the time shift between a pair of range gated echoes by searching for the time shift with the maximum correlation between the RF sampled waveforms. This shift indicates the distance a group of scatterers has moved between pulses. Because the time between pulses is known, the velocity estimate is accurate and self calibrating.

Experiments with a 5 MHz transducer indicate that the standard deviation of the estimate of steady fluid velocity through 6 mm diameter tubes is less than 10% of the mean when the signal-to-noise ratio is 20 dB. Flow rates from 50 to 500 ml/min have been measured with an accuracy better than 10%. Experiments which verify that the time domain correlation method works with whole blood have been completed. The 2-D velocity field is estimated by slowly sweeping the ultrasonic beam across the blood vessel phantom. Total volume flow through the vessel is estimated from the 2-D velocity field. This volume flow estimate is then compared to accurate hydrodynamic flow measurements in order to determine the overall experimental accuracy of the time domain method.

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CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 INTRODUCTION

The measurement of human blood flow by ultrasonic means has proved to be a valuable tool for clinical diagnosis of vascular disease. Unfortunately, current Doppler based measurement techniques are plagued with practical as well as theoretical difficulties which result in inaccurate and imprecise flow measurements.

Doppler blood flow measurement has been well researched from a practical [1-7] as well as a theoretical [8-10] point of view. Unfortunately, the Doppler frequency shift of high frequency ultrasound (typically 7.5 MHz is used clinically) is rather small and hard to measure with useful accuracy. While most investigators have studied the Doppler techniques which were originally used in radar systems [11], another group has chosen to use high speed time domain correlation techniques [12]. Briefly, the time domain correlation technique correlates two successive echoes at a particular range distance from the transducer. The time shift where the maximum correlation occurs is used to determine the distance a section of blood scatterers has moved between the two echoes.

Some important clarifications in terminology are in order at this point. Flow is a term with many definitions. In this thesis, the term volume flow will be used to distinguish flow from flow velocity. Volume flow is the volume of a fluid passing through a given area per unit time. Normally, the area of the confining vessel or tube is used in the flow calculation. The term flow velocity will be reserved for measurements where the direction of the vector quantity, velocity, is important. Thus, axial flow velocity (or just axial velocity) is the component of the flow velocity along the axis of the ultrasound beam. Axial velocity is the velocity which can be measured by most ultrasonic flow measurement techniques including pulsed Doppler and the time domain technique described herein.

Other often confused concepts are accuracy, uncertainty, and precision. Accuracy is a comparison of a measured quantity to the true value obtained from a standard unquestioned experiment. For example, in this research the ultrasonic determination of volume flow will be compared to a highly accurate and independent determination of volume flow, namely, measuring the time required to fill a known volume. Since relatively long times (> 60 s) and large volumes (0.25 liters) can be measured very accurately (within 1%), this hydrodynamic flow measurement is used for the standard reference or true value.

Numerical definitions of accuracy also vary. Experimental inaccuracy (often called error) is often expressed as $X \pm D$ where X is the measured value and D is the "maximum difference" from the measurement and the true value. Since any measurement is

subject to random variations with some probability density function, a "maximum difference" is not a quantitative measure of accuracy unless the probability of the exact value being outside the $\pm D$ limits is zero. In this thesis, accuracy expressed as $X \pm D$ will imply that $\pm D$ is the 95% confidence limits of the measurement. For normally distributed measurement populations, D is approximately two standard deviations. Thus, a stop watch time measurement of 60.0 ± 0.5 seconds implies that 95% of the time the stop watch measurement is accurate to within 0.5 seconds. Another way to express accuracy is as a percent deviation from the true value, that is, "accurate to within 0.83% of 60 seconds." Once again, the deviation must be expressed as a confidence limit; a 95% confidence limit is used in this thesis.

The concept of uncertainty is the degree of unpredictability or lack of confidence in a particular measurement. Uncertainty is also closely related to accuracy in the sense that accuracy is also a way of expressing the confidence of a measurement. Uncertainty will be expressed in the same way as accuracy ($X \pm D$) where D is the 95% confidence interval. The uncertainty of a measurement process is often estimated from a knowledge of the experimental method or skill of the person making the measurement. The accuracy of an experiment is often estimated from the uncertainties of the various measurements needed to obtain an experimental result. The experimental results reported herein are dependent on the uncertainties of the fundamental measurements of time, volume, and distance.

Precision is the extent to which a given set of measurements of a quantity agree with their mean. It is a measure of the repeatability, spread, or variation of the quantity measured. An estimate of the precision of a measurement can be obtained by simply repeating the experiment a number of times and examining the results. In order to quantify precision it is defined numerically as the ratio of the estimate of the standard deviation to the mean of the set of measurements. This ratio will be expressed in percent for convenience. Other authors [10] have called this ratio the "relative standard deviation" and expressed it as a fraction. In both cases, better precision corresponds to a lower number and higher imprecision corresponds to a higher number.

The previous work by Foster [12] considered the one dimensional measurement of flow along an ultrasonic beam passing through a vessel. The experimental and computer simulation results showed that precise estimates of the speed of the scatterers along the beam axis can be obtained. The experimental measurement of volume flow was accomplished by attempting to align the ultrasonic beam so that it passed directly through the center of the vessel. The flow profile was assumed to be parabolic in shape and axially symmetric. Considerable difficulty was encountered in determining the measurement angle and the position where the beam intersected the vessel. Both of these difficulties resulted in quite large experimental errors (greater than 20%) in the volume flow measurement.

This research continues the development of the time domain correlation method and investigates the measurement of the two dimensional flow profile in a blood vessel phantom. From these two dimensional measurements, an accurate volume flow estimate is determined.

1.2 BLOOD FLOW MEASUREMENT BACKGROUND

This section is intended to provide some background on the current Doppler flow measurement methods. No attempt has been made to provide a complete review of Doppler ultrasound or all of the limitations of Doppler flow measurement methods. More detailed reviews of Doppler flow measurement methods can be found in [1,2] as well as [12]. The first blood flow measurements were obtained by using a continuous ultrasound signal; such devices are called CW Doppler flow meters. The CW Doppler flow measurement method will be briefly described in the next section. The primary limitation of CW methods is that no velocity versus range information can be obtained. By using short bursts of ultrasound, pulsed Doppler flow measurement methods can be used to generate velocity versus range data. Pulsed Doppler methods will be considered in section 1.2.2.

1.2.1 CW Doppler

Continuous wave Doppler flowmeters use two transducers to measure the frequency shift in the scattered ultrasound signal. One transducer transmits continuous, single frequency ultrasonic energy, and the other transducer receives the energy scattered

from the blood contained within the vessel as well as all other scatterers within the receive transducer's beam. The difference frequency between the received and the transmitted signal can be used to estimate the mean flow velocity [13-16]. Any moving scatterer in the ultrasound beam will have an influence on the received Doppler spectrum. Since the transducers are generally focused, the region of flow measurement is confined to a reasonably small area. This method of flow measurement is identical to moving target indication (MTI) methods used in radar systems [11] where the radar antenna directivity selects the moving target to be measured. Thus, all moving scatterers within the ultrasonic beam, regardless of range, influence the measured Doppler frequency shift. Since the range of the moving scatterer is unknown, flow versus range measurements are impossible.

Because a single frequency is transmitted, the frequency dependent attenuation of the intervening tissue will not affect the presence of a particular frequency component. The average flow across the vessel measured by CW Doppler is therefore unbiased due to the intervening tissue. However, transit time and beam size effects both bias the flow estimate [13]. Even with these difficulties, the shape of the mean flow estimate versus time from CW Doppler meters has been used to diagnose vascular system defects [1,2,20]. However, since the Doppler angle and blood vessel size are not measured, no quantitative flow information is available.

1.2.2 Pulsed Doppler

Early investigators used long ultrasonic pulses to measure the mean Doppler shift as a function of range [4-7]. Since the variance of the Doppler estimate increases with increasing bandwidth [10], long ultrasonic pulses were required to produce precise flow estimates. Unfortunately, the long pulse length also spatially averaged the flow profile and resulted in poor range resolution of the flow estimate. These pulsed Doppler devices typically consisted of one phase detection circuit for each range gate where the average flow velocity was estimated [5]. Such analog methods were limited to analog frequency domain methods. When inexpensive digital signal processors became available, much of the attention was directed to discrete time spectral estimation techniques [8-10].

Real time flow mapping systems using phased array sector scanners have recently become available [3]. An ultrasonic beam is swept rapidly across the image area. Each line in the sector image is divided into range cells and a flow velocity estimate is determined. The velocity estimate is determined from the Doppler spectrum by a simple autocorrelation technique. The spectral bandwidth and mean frequency of the Doppler spectrum are color coded and displayed with the gray scale image of the surrounding tissue. Flow toward the transducer is coded as red and flow away from the transducer is coded as blue. Since only the direction of the flow is displayed, no accurate flow rate information is provided by this system.

1.3 RESEARCH PURPOSE

The purpose of this research is twofold. First, the measurement of flow in one dimension along the beam axis will be refined and generalized in order to improve the precision of the time domain correlation method when a wide range of flow velocities is present. Second, a method will be developed to accurately measure the volume flow in a typical vessel. Flow measurement in a plane across the vessel will be used to determine the measurement angle and the total volume flow. The accuracy of this ultrasonic volume flow measurement method will be determined experimentally by comparison with independent hydrodynamic flow rate measurements.

Some of the theoretical and practical problems with pulsed Doppler ultrasound flow measurement are described in Chapter 2. The time domain correlation flow measurement technique is discussed in Chapter 3. The equipment and methods used in the flow measurement experiments are discussed in Chapters 4, 5, and 6. The experimental results, discussion and conclusions are presented in the remaining chapters.

CHAPTER 2

PULSED DOPPLER ULTRASOUND

This chapter is intended to provide background information about pulsed Doppler flow measurement and indicate some important drawbacks of Doppler frequency domain approaches to flow measurement. Considerable interest in blood flow imaging is indicated by the sophistication of blood flow measurement devices which have recently become available [3]. It is therefore important to understand and determine the quantitative limitations of Doppler flow measurement methods. The following discussion addresses the two primary limitations of Doppler blood flow measurement: accuracy and precision. In later chapters the accuracy and precision of the time domain correlation method will be compared to the values obtained for Doppler flow measurement.

One of the major accuracy problems with the current Doppler flow measurement techniques is that the volume flow is estimated, not measured [21]. The typical Doppler flowmeter measures the average flow velocity along the beam direction at a particular distance from the transducer. Both the blood vessel size and the measurement angle are unknown. The vessel size is estimated from a knowledge of an average size for the particular vessel being measured or by pulse-echo scanning methods [1,2]. Some

researchers have suggested using two or more different measurement positions (by moving the transducer or by having more than one transducer) and thus determining the Doppler angle by using a number of range estimates of the peak flow [7,16]. Since arterial vessel walls are constantly moving (due to the pulsatile flow of blood), it is difficult to determine the position of the vessel at a given time. Also, since the paths for each measurement are different, the two measurements could have different biases due to different tissue effects. Since each patient has different size blood vessels and it is difficult to measure the Doppler angle with precision, quantitative comparisons between normal and abnormal flow are nearly impossible.

As shown in [10] the typical pulsed Doppler flow meter measures the mean frequency in the echo. The optimum correlation receiver method of Doppler flow measurement suggested in [31] measures the frequency of the component with the largest amplitude. In [31] a number of correlation receivers are shown to be the optimum Doppler frequency shift detector in the maximum likelihood sense. Other devices such as the zero crossing detector can be used to estimate the Doppler shift. For large signal-to-noise ratios the zero crossing detector has been shown to follow the root mean square frequency of the return echo [32]. Since most Doppler flow meters which are currently available measure the mean frequency in the received echo, the mean frequency will be considered in all simulations presented in this chapter.

The bias of the mean Doppler frequency estimate due to the frequency dependent attenuation of tissue will be used in the next section to demonstrate one accuracy limitation of Doppler flow velocity measurements. Section 2.2 addresses the other major limitation of Doppler flow measurement: precision. By reviewing several theoretical and experimental results presented in the literature, the precision of typical Doppler blood flow axial velocity measurements will be determined. The Doppler precision values obtained from the literature will be compared to the time domain correlation precision results in Chapter 7 (see section 7.1.4).

2.1 ERRORS INTRODUCED BY TISSUE ATTENUATION

Most current clinical blood flow measurement techniques are based on qualitative comparisons of measurements from the same patient [16-21]. This is primarily because the variation of the vascular system from patient to patient can be significant. The thickness and characteristics of the intervening tissue between the skin surface and the flow to be measured affects the accuracy of Doppler frequency estimate. Different tissues have different frequency dependent attenuations [22-29] and will therefore change the measurement of the mean frequency of the received echo which is used to estimate the Doppler shift [30]. The presence of more tissue between the transducer and the flowing blood in a particular patient may not only increase the variance of the Doppler frequency estimate (because of a lower signal-to-noise ratio due to a longer tissue path length) but may also bias the

estimate in an unpredictable way. This is because the received ultrasonic echo spectrum can have a different power spectrum than that of the transmitted signal.

To determine the effect of frequency dependent attenuation on a Doppler flowmeter output, this simulation assumes that the flow meter measures the mean frequency of the ultrasonic backscattered signal. Gerzberg [33] has shown that the pulsed Doppler spectral estimate is biased due to a finite signal-to-noise ratio. In this simulation, it is also assumed that no noise is present in the received echo. For simplicity, the blood motion is modeled to be moving at a constant, uniform velocity toward the transducer. An approximation valid for long pulses ($M \geq 5$) of the spectrum of the pressure signal is given by [35],

$$T(f) = H(f) \text{Sinc} \left(M \left(\frac{f}{f_0} - 1 \right) \right) \quad (1)$$

$$H(f) = \frac{f f_0}{Q \sqrt{(f^2 - f_0^2)^2 + (f f_0/Q)^2}} \quad (2)$$

where

- $T(f)$ = spectrum of transmitted pressure waveform.
- $H(f)$ = transducer second order transfer function.
- $\text{Sinc}(f) = \sin(\pi f)/\pi f$.
- M = number of cycles of center frequency excitation.
- f_0 = center frequency (Hz).
- Q = transducer quality factor.

A typical transmitted signal spectrum ($T(f)$) with $M=5$ is shown in Figure 1. The magnitude in decibels is shown for a center frequency of 5 MHz. In this simulation the transducer is

modeled as a second order bandpass filter with a Q of 3. This transmitted signal spectrum, $T(f)$, will be used throughout the simulation to demonstrate the errors introduced by the tissue attenuation.

A suitable tissue model determined by several investigators [22-29] with CW measurements of many tissues is given by

$$\text{atten}(f) = A_0 f^b \quad (3)$$

where

$\text{atten}(f)$ = attenuation of ultrasound as a function of frequency (dB).

A_0 = attenuation in dB at 1 MHz (typically 0.5-2 dB).

f = frequency in MHz.

b = frequency independent constant (typically 1-1.4).

The distance traveled by the ultrasonic beam is included in A_0 . If a_0 is the attenuation coefficient (dB/cm, for example), then at 1 MHz A_0 is $a_0 d$ for a one way tissue path length d . The value of a_0 for various materials varies from nearly zero for water to 1 dB/cm for some soft tissues [26-29]. For peripheral vascular blood flow measurements, the one way path length may be less than 1 cm. Thus, in many soft tissue cases, reasonable values of A_0 are between 0.5 dB and 1.0 dB. The value of b varies from 1.0 to 1.4 for many soft tissues [27,28]. The magnitude of the pressure signal which is incident at the blood vessel is

$$B(f) = T(f) 10^{-\frac{\text{atten}(f)}{20}} \quad (4)$$

As the ultrasonic wave passes through the tissue the mean ultrasonic frequency is shifted. When the ultrasonic wave reaches the moving scatterers, each of the frequency components in the scattered waveform has been shifted according to the Doppler relation,

$$f_{\text{scat}} = f \left(1 - \frac{2V}{c} \right) \quad (5)$$

where

- f_{scat} = frequency backscattered by blood.
- f = frequency incident on blood.
- V = blood speed (positive is toward the transducer).
- c = speed of sound in blood.

As the wave propagates back through the tissue, it is again attenuated according to equation (3). The transducer then filters the received pressure signal. The received electrical signal, $R(f)$, is

$$R(f_{\text{scat}}) = H(f_{\text{scat}}) B(f_{\text{scat}}) 10^{-\frac{\text{atten}(f_{\text{scat}})}{20}} \quad (6)$$

The magnitude of each of these signals is shown in Figure 1 for $A_0 = 1$ dB and $b = 1.4$. These values are typical for soft tissues (liver, for example) [27,28]. The axial flow velocity for this case is 10 m/s and $c = 1500$ m/s. For the case illustrated, the mean frequency which is incident at the scattering target is 4.933 MHz (see the curve labeled "AFTER

TISSUE FORWARD TRANSIT" in Figure 1). The tissue has caused a -66.67 kHz shift in the transmitted spectrum (mean 5.000 MHz). The Doppler shift caused by the 10 m/s scatterer motion in this case is -66.66 kHz as determined from equation (5). This shift is indicated in Figure 1 for the curve labeled "DOPPLER SHIFTED." During the reverse transit back to the transducer the tissue causes a further -66.65 kHz shift in the mean frequency of the ultrasonic signal (see the curve labeled "AFTER TISSUE REVERSE TRANSIT"). The received echo signal after the transducer filtering (see the curve labeled "RECEIVED SIGNAL") has a mean frequency shift of -153 kHz. Thus, a Doppler shift of -66.66 kHz results in a mean frequency shift of -153 kHz in the received echo signal. The example shown in Figure 1 illustrates the method used to simulate the effects of the tissue attenuation.

Figure 2 is a graph of the mean frequency shift of the received echoes for a 10 cycle transmitted pulse and $b = 1.4$. Three different A_0 values ($A_0 = 0.0, 0.5, 1.0$) are shown to illustrate the relationship between target velocity and mean frequency shift of the received echoes. This figure shows a linear relationship between frequency shift and target velocity. The bias for $A_0 = 1.0$ is -40 kHz indicates a bias greater than the shift caused by an axial velocity of 6 m/s. At 5 MHz the shift without tissue effects for $V = 6$ m/s would be 40 kHz. The slopes of all of the frequency shift curves are equal and have a magnitude less than $2 f_0/c_0$, which is what would be expected from equation (5). This is caused by the slight filtering by the transducer ($Q = 3$) of the ultrasonic signal spectrum. After the

frequency spectrum is Doppler shifted, the transducer filtering causes the mean received frequency to be closer to the center frequency of the transducer. Because of the transducer filtering, if the bandwidth of the excitation is large compared to the bandwidth of the transducer, the slope will approach $-f_0/c_0$. If the number of cycles is large compared to the Q of the transducer (ten or greater in this case), the slope is determined by equation (5). The bias of the frequency shift curve is determined by the tissue parameters.

Figure 3 shows the frequency shift of the received echo signal versus target velocity for three different b values ($b = 1.0, 1.2, \text{ and } 1.4$). Ten cycles of excitation ($M = 10$) is simulated for $A_0 = 0.5$. Again, a linear relationship with target velocity is shown. The bias of the mean frequency shift changes with the tissue parameter b . The bias due to $b = 1.0$ is only -9 kHz and increases to -23 kHz for $b = 1.4$.

Figure 4 shows the bias of the mean frequency shift versus the number of cycles for three different A_0 values with $b = 1.4$. The upper most curve for $A_0 = 0.0$ indicates no bias when there are no tissue effects. The other two curves show the bias of the mean frequency shift for $A_0 = 0.5$ and 1.0 . The mean frequency bias of the received echoes decreases with decreasing A_0 . Since the tissue path length is included in A_0 , a shorter tissue path length will have a smaller mean frequency bias. Also, a larger number of transmitted cycles results in a smaller transmitted bandwidth resulting in a less severe mean frequency bias due to the tissue path.

Figure 5 shows the bias of the mean Doppler frequency estimate versus the number of cycles for $b = 1.0, 1.2,$ and 1.4 with $A_0 = 0.5$. For these cases, the bias due to the tissue decreases as the number of transmitted cycles increases and the transmitted bandwidth decreases. This is consistent with the CW case where a single frequency, zero bandwidth signal is used. Even when 20 cycles are used, the bias is still -4 kHz for $A_0 = 0.5$ and $b = 1.0$.

These simulation results show that a large bias results when the mean Doppler frequency is used to estimate the blood flow velocity. Even if 20 cycles of the 5 MHz transmitted signal were used with $A_0 = 0.5$ and $b = 1.0$, the bias in the mean frequency estimate would be -4 kHz. If the axial velocity was -2 m/s (a rather large blood velocity), then the Doppler shift would be 13.33 kHz. Because of the -4 kHz bias, the received echo signal would have a 9.33 kHz frequency shift. Thus, if the frequency shift of the received echoes were used to indicate an axial velocity of -2 m/s the resulting velocity estimate would be -1.4 m/s, assuming the tissue attenuation model given by equation (3). The poor accuracy of this example Doppler axial velocity measurement indicates the large error (30% in this case) in the Doppler flow velocity estimate which may be introduced by the frequency dependent attenuation of typical soft tissue.

2.2 DOPPLER ESTIMATE PRECISION

This section describes several theoretical and experimental results presented in the literature which indicate the precision of Doppler axial flow velocity measurements. Although most authors do not include information about the precision of Doppler flow measurements, one recent publication has addressed this issue exclusively [34]. For purposes of comparison with the time domain correlation method, the Doppler flow estimate precision will be defined as the precision of a flow estimate obtained from one received echo. The reported precision of a Doppler flow meter includes the averaging effect of the output lowpass filter or integrator. Proper comparison of Doppler results to the time domain correlation method requires compensation for the effect of these output filters. Often only the -3 dB frequency of analog filters is stated, making the modeling of the output filter difficult. Theoretical results [9,34] are easier to compare since the averaging time is usually stated. Other theoretical results [31] suggest a different approach to the measurement of the Doppler frequency by using a series of correlators.

The maximum correlation method suggested by Olinger [31] used a number of correlation receivers all set at different frequencies. The correlation receiver with the largest output was used to determine the most likely Doppler shift. Olinger shows that as the number of correlation receiver elements increases above an optimum number, the correlator with the maximum output was equally likely to be either of the adjacent frequencies. Olinger states that three correlators are a

reasonable number for the typical transducer configuration. Thus, the flow rate was represented by three quantization levels.

Let the correlator set to the transmit frequency be correlator 0, the middle correlator set to a 1 kHz shift be correlator 1, and correlator 2 be set to a 2 kHz shift. Suppose the frequency shift to be measured was half way between the frequency of correlators 1 and 2. The correlator with the greatest output is equally likely to be correlator 1 or 2. The probability mass function for this case is 0, 0.5, 0.5 for correlators 0, 1, and 2 respectively. The mean flow velocity measured by the levels is 1.5 as desired. The variance is 0.25 for this simple example. The precision of this discrete flow estimate is therefore 33%. This theoretical result for an optimum bank of correlators indicates the approximate precision of the best Doppler blood flow measurement methods.

Angelsen and others have published results concerning the precision of the Doppler flow estimate. The continuous time estimators of the mean frequency of a pulsed Doppler spectrum are considered in [10] and discrete time methods are considered in [9,34]. In both cases, the output of the mean Doppler frequency estimator is low-pass filtered before the standard deviation is calculated. For a typical case in [10] with a low-pass filter with two poles at 20 Hz and a parabolic flow profile, the theoretical precision is 9%. Experimental data in [10] show the precision was 11% for the same conditions. The discrete time implementation using a single pole 15 Hz low-pass filter [9] has a theoretical precision of approximately 10% for Doppler shifts

greater than 600 Hz. Several theoretical mean frequency estimators are compared in [34]. Fifty outputs of all these estimators are averaged together. The precision of all the estimates is 10-12% for a 10 dB signal-to-noise ratio. In all cases compared, a pulse repetition frequency of 4000 Hz was chosen and the angle between the ultrasonic beam and the flow was 45°. A large number of Doppler frequency estimates are averaged together to obtain the given precision. An approximate expression of the averaging time caused by a first order low-pass filter is [35],

$$T_{ave} = \frac{2}{\pi f_3} \quad (7)$$

where f_3 is the cutoff frequency (Hz) of the filter. Filters of higher order have a longer averaging time, so this is an optimistic estimate of the averaging time. Because the received echoes are known to be approximately Gaussian, the output of the Doppler flow meter can be considered Gaussian [10]. Since the flow rate being considered in this comparison is constant over the measurement time interval, the Doppler flow meter output can also be assumed to be a wide sense stationary process [10]. The standard deviation of the output of the filter is reduced by the factor $1/\sqrt{M}$ where M is the number of input samples averaged together by the filter.

The three cases indicated above [9,10,34] can now be compared to other flow measurement methods by determining the standard deviation of the unfiltered flow estimate. The results are shown in Table 1.

Table 1 - Doppler Flow Estimate Precision.

Case	Precision Reported (%)	Filter Cutoff (Hz)	Average Time (sec)	Number Averaged (M)	Unfiltered Precision (%)
Theory [10]	9	16	0.04	170	117
Experiment [10]	11	20	0.03	127	124
Discrete Theory [9]	10	15	0.04	170	130
Discrete Theory [34]	12	51	0.01	50	85

The best unfiltered Doppler precision listed in Table 1 for a discrete theoretical result [34] is 85%. This result did not include the effects of velocity gradients in the analysis. The other results [9,10] did include some velocity gradient effects in the Doppler flow estimate precision. Since a range of axial flow velocities is present in a typical flow measurement region, the precision values for the other three cases shown in Table 1 are considerably larger (117 to 130%) and may be more typical of Doppler blood flow measurement.

In summary, the approximate unfiltered precision of typical Doppler axial flow velocity measurements has been determined for theoretical and experimental cases published in the literature. The unfiltered precision is the precision of consecutive Doppler frequency shift estimates obtained from each echo. The unfiltered precision of Doppler will be compared to the precision of the time domain correlation method in Chapter 7 (section 7.1.4).

CHAPTER 3

TIME DOMAIN FLOW MEASUREMENT

3.1 TIME DOMAIN CORRELATION

The measurement of distance by the time of arrival of electromagnetic energy has been used in radar for many applications [11]. Early radar systems used a threshold detector to indicate the position of a target on the radar screen. Later in the development of radar, optimal time domain correlation or "matched filtering" was suggested as the best way to estimate the time of arrival of the radar pulse [11]. If a radar antenna is made to track a target (as it is in fire control radar [11]), the change in the time of arrival of the received echoes can be used to determine the axial velocity of the target. The essence of the time domain flow measurement technique is to track scatterers within an ultrasonic beam and determine how far they move by the time shift between the echoes.

Because the ultrasonic beam is fixed in space and can not follow the flow, the time domain method requires that the time between echoes must be made sufficiently small so that most of the scatterers stay within the beam and the correlation between the two echoes remains high. The time shift between one echo and the next can therefore be estimated by searching for the time

shift which yields the maximum correlation. The time domain correlation method estimates the time shift between two successive echoes in order to determine the amount the scatterers have moved during the time between the two echoes.

Other investigators [36] have suggested using time domain correlation to measure flow. However, their work used two different transducers separated by a small distance along the tube. Although the technique used in [36] was shown to have an accuracy of $\pm 2\%$ when relatively fast slurry flow was being measured, it would be difficult to physically displace two identical transducers by a small enough amount (1-5 mm) in order to still have a high correlation between the two received signals from the slow blood flow in veins and the smaller arteries. Also, blood vessels are not straight, resulting in different angular positions that the two beams would intersect the vessel.

This section discusses the time domain correlation method by using a set of ultrasonic measurements to illustrate the time domain correlation method. Further details of the time domain correlation method and simulation results illustrating many of the concepts described in this chapter can be found in the previous work [12].

Figure 6 shows three consecutive ultrasonic echoes measured using a 5 MHz transducer directed at a 6 mm diameter dialysis tube containing flowing water and Sephadex^R. The time between these echoes was 0.384 ms (PRF=2604 Hz) and the angle between the ultrasonic beam and the flow velocity was approximately 45°. The

one way path length traveled by the ultrasonic beam from the front wall to the back wall was approximately 8.5 mm. The round trip time for the ultrasonic wave to travel across the tube was approximately 13 μ s as indicated in Figure 6 by the length of the received ultrasonic signal. Careful examination of these echoes reveals that the echoes look very similar. Also, the center sections of the echoes appear to move to the right slightly and echo sections near the wall positions do not move.

The lower portion of Figure 6 illustrates the time domain correlation method used to determine the amount of motion in three 0.8 μ s range gated sections of the three echoes. The discrete time correlation between two zero mean echo sections, $e_1(n)$ and $e_2(n)$ is given by

$$\rho(s) = \frac{\sum_{i=0}^{N-1} e_1(r+i) e_2(r+i+s)}{\sqrt{\sum_{j=0}^{N-1} [e_1(r+j)]^2 \sum_{k=0}^{N-1} [e_2(r+k+s)]^2}} \quad (8)$$

where

s = shift between echoes (samples).

r = range to beginning of section to be measured (samples).

$\rho(s)$ = correlation between echoes (-1 to 1).

N = correlation length.

Each pair of the echo sections is correlated resulting in the nine sample correlation functions shown. In the sample correlation functions shown in Figure 6, the correlation length was 40 samples. The sampling rate was 50 MHz so that 40 samples corresponds to 4 wavelengths at 5 MHz. The work by Foster [12]

indicated that this correlation length is a good compromise between range resolution and measurement precision.

As the correlation examples in Figure 6 show, the maximum correlation of the center section of the echo is at a positive time shift. This corresponds to flow velocity away from the transducer. The correlation peak between echo 1 and echo 3 is at a shift which is twice that of echo 1 and echo 2. The other two correlation examples for echo sections near the tube walls show less shift, as expected. The proposition introduced in [12] is that the velocity of the flowing fluid is given by

$$V = \frac{E[s_{\max}] c}{2 T \cos(\theta)} \quad (9)$$

where

- V = magnitude of the velocity of the scatterer at a particular range.
- s_{max} = time shift with maximum correlation.
- E[s_{max}] = expected value (mean) of time shift.
- T = time interval between the transmitted pulses.
- c = speed of sound in the scattering fluid.
- θ = angle between sound beam and velocity vector (measurement angle).

Since the time domain correlation method compares two successive echoes which have both passed through the same intervening tissue, frequency dependent attenuation of the tissue path will not affect the mean flow measurement. Considering the effects on the frequency spectrum due to typical tissues, this is a major advantage of the time domain correlation technique. Thus, the average time shift determined by time domain correlation is unbiased by the frequency response of the tissue. This will be shown in the next section along with the derivation of the variance of the time domain correlation method.

3.2 PRECISION OF TIME DOMAIN CORRELATION METHOD

Foster's work [12] measured the precision of the time domain correlation flow estimates experimentally for a particular transducer configuration. These experimental precision values illustrate the importance of understanding the factors which influence the variance of the time domain correlation flow estimates. Table 2, which was adapted from a graph in [12], shows the effect of changing the time between echoes:

Table 2 - Precision and Relative Variance for a 5 MHz Transducer Measuring Buttermilk Flow at a 45° Measurement Angle (adapted from [12]).

Shift (wavelengths)	Precision (%)	Relative Variance
0.125	12	5.8
0.250	10	4.0
0.375	8	2.6
0.500	6	1.4
0.625	5	1.0
0.750	5	1.0
0.875	7	2.0
1.000	10	4.0
>1.000	>10	>4.0

The first column shows the shift in wavelengths between the two consecutive echoes. The same midstream velocity was used in order to minimize the other errors due to the finite beam size and the flow velocity gradient in the experimental measurement. The data shown in Table 2 were obtained with a 5 MHz transducer having a beamwidth of 2 wavelengths at the focal point (49 mm from the transducer front surface). The relative variance versus shift curve depends on the transducer beamwidth and the measurement angle (45° for the above data). For each shift, the pulse repetition rate was changed until the mean shift was the

value shown. The precision of the time shift (the second column) was then calculated from the standard deviation and mean time shift values obtained from the graph in [12]. The last column shows the variance of the estimate relative to the minimum precision (5% at 0.625 wavelengths). Each precision value was divided by 5% and then squared to obtain the relative variance values. More detailed measurements of the experimental precision of the time domain correlation method are presented in Chapter 7. Although experimental results can show trends in the measurement precision, it is difficult to establish the relationship between the parameters which change the precision value.

The remainder of this section is a theoretical derivation of the precision of the time domain correlation method. By making certain approximations about the statistical properties of the ultrasonic echoes, a relationship is established between the measurement precision and the transducer characteristics, measurement angle and signal-to-noise ratio.

This derivation of the precision of the time domain method is almost identical to the derivation of the time of arrival of a radar signal presented by Helstrom [37]. A similar radar result was also found by Woodward [38]. The overall assumption in this derivation is that the signal level is large compared to the noise level. The precision derivation assumes that the correlation length is infinite and that the correlation peak can be estimated from a continuous correlation estimate. Many of the simulation results in [12] addressed the validity of these assumptions and showed that a four wavelength long discrete

correlation had little additional variance than infinite continuous correlations.

Consider two echoes,

$$\begin{aligned} r_1(t) &= f_1(t - \tau_0) + n_1(t), \\ r_2(t) &= f_2(t) + n_2(t), \end{aligned} \quad (10)$$

where $f_1(t)$ is the backscattered signal from a moving fluid at one instant in time and $f_2(t)$ is the signal from the same region of the fluid at a latter time T . The time shift to be estimated is τ_0 . $n_1(t)$ and $n_2(t)$ are samples of uncorrelated, zero mean, Gaussian random processes due to the radio frequency interference and electrical noise present in the measurement system. The correlation function which is maximum near $\tau = \tau_0$ is given by

$$\begin{aligned} G(\tau) &= \int_{-\infty}^{\infty} r_2(t - \tau) r_1(t) dt \quad (11) \\ &\approx \int_{-\infty}^{\infty} [f_2(t - \tau) f_1(t - \tau_0) + n_2(t - \tau) f_1(t - \tau_0) + n_1(t) f_2(t - \tau)] dt \quad (12) \end{aligned}$$

The $n_1(t)n_2(t)$ term was neglected due to the fact that it is quadratic in noise and will therefore be much smaller than the signal components. $G(\tau)$ can be written as follows:

$$G(\tau) = h(\tau) + m(\tau) \quad (13)$$

where

$$h(\tau) = \int_{-\infty}^{\infty} f_2(t - \tau) f_1(t - \tau_0) dt \quad (14)$$

$$m(\tau) = \int_{-\infty}^{\infty} n_2(t - \tau) f_1(t - \tau_0) + n_1(t) f_2(t - \tau) dt \quad (15)$$

Note that $h(\tau)$ is the deterministic portion of $G(\tau)$ and $m(\tau)$ is subject to random variation due to the noise components in the received echoes. Since $n_1(t)$ and $n_2(t)$ are zero mean Gaussian random variables, the maximum of $G(\tau)$ will occur near $\tau = \tau_0$ with an expected value $G_0(\tau_0)$ defined by

$$G_0(\tau_0) = h(\tau_0) = \int_{-\infty}^{\infty} f_2(t) f_1(t) dt \quad (16)$$

The amount that the noise terms $n_1(t)$ and $n_2(t)$ change the location of the maximum correlation can be modeled as a Gaussian random variable and characterized by its variance and mean. By setting the derivative of $G(\tau)$ equal to zero, the maximum correlation solution can be determined, that is,

$$G'(\tau) = h'(\tau) + m'(\tau) = 0 \quad (17)$$

Because $n_1(t)$ and $n_2(t)$ are zero mean random variables, the solution for τ in (17) will be near τ_0 [37]. To determine the error in the estimate of the time shift, the equation is expanded in powers of $(\tau - \tau_0)$. Keeping first order terms and using the fact that $G'(\tau_0) = 0$ [37], gives the result,

$$(\tau - \tau_0) G''(\tau_0) + m'(\tau_0) = 0 \quad (18)$$

so that

$$\tau - \tau_0 = \frac{- \frac{d}{d\tau} [f_1(t + \tau - \tau_0)n_2(t) + f_2(t - \tau)n_1(t) dt] \Big|_{\tau = \tau_0}}{G''(\tau_0)} \quad (19)$$

Since $n_1(t)$ and $n_2(t)$ are zero mean random variables, the estimate of τ_0 is unbiased. The denominator is a constant for a particular measurement system. Since $n_1(t)$ and $n_2(t)$ are independent, the numerator is a random variable composed of two independent parts. The variance is given by

$$\text{Var}[\tau - \tau_0] = \frac{\text{Var} \left[\int_{-\infty}^{\infty} h_1(t)n_2(t) dt \right] + \text{Var} \left[\int_{-\infty}^{\infty} h_2(t)n_1(t) dt \right]}{[G''(\tau_0)]^2} \quad (20)$$

where

$$h_1(t) = \frac{d}{d\tau} f_1(t + \tau - \tau_0) \quad (21)$$

$$h_2(t) = \frac{d}{d\tau} f_2(t - \tau) \quad (22)$$

Using well known results for the response of a linear system to a Gaussian noise source [37], the variance can be expressed in terms of frequency domain quantities as

$$\text{Var}[\tau - \tau_0] = \frac{\frac{1}{2\pi} \int_{-\infty}^{\infty} [|H_1(\omega)|^2 + |H_2(\omega)|^2] \eta(\omega) d\omega}{[G''(\tau_0)]^2} \quad (23)$$

where

$$H1(\omega) = j\omega e^{j(\tau - \tau_0)} F1(\omega) \quad (24)$$

$$H2(\omega) = -j\omega e^{-j\omega\tau} F2(\omega) \quad (25)$$

$\eta(\omega)$ = power spectrum of $n1(t)$ and $n2(t)$

Equation (23) can be simplified as follows:

$$\text{Var}[\tau - \tau_0] = \frac{\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 [F1^2(\omega) + F2^2(\omega)] \eta(\omega) d\omega}{\left[-\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 F2^*(\omega) F1(\omega) d\omega \right]^2} \quad (26)$$

For white noise $\eta(\omega) = N_0$ so that

$$\text{Var}[\tau - \tau_0] = \frac{\frac{N_0}{2\pi} \int_{-\infty}^{\infty} \omega^2 [F1^2(\omega) + F2^2(\omega)] d\omega}{\left[-\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 F2^*(\omega) F1(\omega) d\omega \right]^2} \quad (27)$$

$$\text{Var}[\tau - \tau_0] = \frac{\frac{N_0}{2\pi} \int_{-\infty}^{\infty} [F1^2(\omega) + F2^2(\omega)] d\omega}{(\overline{\omega^2} - \bar{\omega}^2) \rho^2 E1 E2} \quad (28)$$

where

$$\rho = \frac{\int_{-\infty}^{\infty} f2(t) f1(t) dt}{E1 E2} \quad (29)$$

and the energy of each echo is defined as follows:

$$E1 = \int_{-\infty}^{\infty} f1^2(t) dt \quad E2 = \int_{-\infty}^{\infty} f2^2(t) dt \quad (30)$$

The RMS bandwidth of the received echoes, β , is defined as

$$\beta^2 = \overline{\omega^2} - \bar{\omega}^2 = \frac{\int_{-\infty}^{\infty} [\omega^2 F^2(\omega) - F^2(\omega)] d\omega}{\int_{-\infty}^{\infty} F^2(\omega) d\omega} \quad (31)$$

and ρ is the correlation coefficient as defined in (29). By applying Parseval's relation on the numerator of (28), the variance of the shift estimate can be simplified to

$$\text{Var}[\tau - \tau_0] = \frac{N_0 (E_1 + E_2)}{\beta^2 \rho^2 E_1 E_2} \quad (32)$$

Since each echo travels through the same medium and the echoes are highly correlated, the two energies are approximately equal, so that $E \cong E_1 \cong E_2$. The precision of the time shift estimate (τ) is therefore

$$\text{Precision} [\tau] = \frac{1}{\beta \rho \tau_0 \sqrt{\frac{E}{2 N_0}}} \quad (33)$$

Figure 7 shows the precision of the time shift estimate versus the mean time shift (τ_0) for the case where $\rho = 1$. This is the case when the two signals, $f_1(t)$ and $f_2(t)$, are identical waveforms (with possibly different energies). The signal-to-noise ratio is $\sqrt{E/N_0}$ in equation (33). As indicated by equation (33), the precision of the time shift estimate is inversely related to the product of the signal-to-noise ratio and the RMS bandwidth. The precision improves with increasing bandwidth or increasing signal-to-noise ratio. Signal-to-noise ratios of 10 dB, 20 dB and 30 dB for an RMS bandwidth (β) of

2.5 MHz are illustrated in Figure 7. The precision value for each time shift improves by an order of magnitude when the signal-to-noise ratio is increased from 10 dB to 30 dB. A similar improvement would also be observed if the RMS bandwidth were increased by an order of magnitude. Figure 7 suggests that the precision of the time shift estimate can be improved by simply increasing the pulse repetition period (T) resulting in a larger mean time shift for the same axial flow velocity. Although the precision continues to improve with τ_0 when the correlation coefficient is unity, the motion of the scatterers through the ultrasonic beam changes the correlation coefficient and the resulting precision relationship.

As more new scatterers enter the beam during the time between echoes, the maximum correlation of the consecutive echoes decreases. For a given scatterer speed, the maximum correlation will decrease as the time between echoes increases and the precision of the velocity estimate will decrease. On the other hand, if the time between echoes decreases to a very small value, the point of maximum correlation and the corresponding velocity estimate will vary randomly due to the random noise in the backscattered signal.

Foster [12] measured the correlation coefficient for a 5 MHz transducer (25.4 mm aperture) and also derived an approximate expression for the correlation coefficient for a cylindrical beam. In both cases a nearly linear decrease in the correlation function with scatterer motion is observed. From measurements and the cylindrical theory, when the scatterers have moved into

half of the beam the correlation coefficient was reduced from 1.0 to 0.3. Thus, according to the results in [12], a good approximation for the correlation between echoes is

$$\rho(a) = 1 - 1.4 a$$

where

$$a = \frac{V T \sin(\theta)}{BW} \quad 0 < a < 5/7 \quad (34)$$

and BW is the 3 dB beamwidth of the transducer at the measurement range and $VT \sin(\theta)$ is the distance the scatterers move perpendicular to the ultrasonic beam axis. The measurement angle (θ), defined in equation (9), changes the correlation between echoes greatly. At $\theta = 0^\circ$ the same scatterers are always in the beam and the correlation coefficient is always unity (if beam effects are ignored). In this somewhat impractical case (the beam is inside the vessel), the precision of the time shift estimate is as illustrated in Figure 7. Figure 8 shows the precision versus time shift between echoes using the above model for the correlation between echoes. When equations (9), (33) and (34) are combined, the precision of the time domain correlation method is as follows:

$$\text{Precision}[\tau] = \frac{\sqrt{2}}{\beta \tau_0 \sqrt{\frac{E}{N_0}} \left(1 - \frac{1.4 \tau_0 c \tan(\theta)}{2 BW} \right)} \quad (35)$$

Figure 8 shows equation (35) as a function of τ_0 for measurement angles of 45, 60 and 75° for a 20 dB signal-to-noise ratio. A beamwidth (BW) of two wavelengths (0.6 mm at 5 MHz) and an RMS bandwidth of 2.5 MHz were used to generate Figure 8.

These values are typical for the experimental measurements with the 5 MHz transducer to be used in many of the flow measurements discussed in Chapter 7. Figure 8 shows that for each measurement angle the precision value decreases to a minimum (about 5% at 45°) and then begins to increase due to the decorrelation of the echoes. The minimum precision value increases with increasing measurement angle, and the time shift with the minimum precision value decreases. This measurement precision could be expected at midstream where velocity gradients within a range cell are small (a variation of less than 1%) compared to the measurement precision indicated with no velocity gradient. Near the vessel walls, the wide range of flow velocities within a range cell make the precision of the velocity estimate much larger than that indicated in Figure 8.

In summary, the theoretical precision of the time domain correlation method depends on the measurement angle, the system RMS bandwidth, the transducer beamwidth, the mean time shift between echoes, and the signal-to-noise ratio. In Chapter 7, the experimental precision of midstream flow measurements at different measurement angles will be compared to the theoretical precision curves shown in Figure 8.

CHAPTER 4

EQUIPMENT

4.1 BLOOD VESSEL PHANTOM

Figure 9 shows a schematic diagram of the blood vessel phantom system. It consists of a temperature controlled water bath (Excal model EX-500), peristaltic pump (Masterflex model 7520) and fluid flow regulator system. Dialysis tubing with an inside diameter of approximately 6.3 mm was used to mimic the blood vessel in order to minimize the effects of the tube on the ultrasonic beam.

4.1.1 Fluid Flow Regulator

The flow regulator shown in Figure 9 works by maintaining a constant difference (or height, H) in the fluid level in the upper and lower glass reservoirs. Any overflow above the top of inner cylinders of the upper and lower reservoirs is returned back to the pump reservoir. The height of the upper reservoir is adjusted to set the head lost by the fluid (H). After a period of time the flow velocity reaches a steady state value where the viscous losses in the fluid equal the potential energy gained by the fluid. This equilibrium flow can be determined from Poiseuille's law [39] for laminar flow.

Laminar flow occurs when each streamline flows independently with no eddies or turbulence [39]. Laminar flow can be steady or pulsatile. Pulsatile laminar flow is characterized by periodically varying flow velocities at each streamline [39]. Steady laminar flow in a fixed size pipe has a constant velocity at each point in the tube. This property of steady flow makes it easier to measure and quantify. It will therefore be used for all ultrasonic volume flow measurements. The constant laminar flow rate [39] is given by

$$Q = \frac{D^4 g \rho_o H}{128 \mu L} \quad (36)$$

where

- Q = laminar flow rate (m³/s).
- D = tube diameter (m).
- g = acceleration of gravity (9.8 m/s²).
- ρ_o = fluid density (kg/m³).
- H = head loss (m).
- μ = viscosity of fluid (N-s/m²).
- L = total tube length (m).

For water (viscosity = 0.001 N-s/m²), a tube diameter of 6 mm and the 1.9 m total tube length (L), a 2 cm head gives a 197 ml/min flow rate. For smaller flow rates (less than 100 ml/min) an extra 2 m long 3 mm diameter plastic tube is added just after the upper reservoir in series with the tube. The larger head required for this long 3 mm tube provides better control and regulation of the flow rate. Since the flow rate is partially determined by the temperature dependent viscosity of the fluid, the temperature is maintained at 25°C throughout the experiment by the water bath heater.

The volume flow rate is measured by closing the return valve on the lower graduated reservoir and measuring the time to fill a given volume. The volume of the outside portion of the graduated cylinder is 253 ml. The time to fill this volume can be measured with an accuracy of approximately ± 0.5 seconds. Thus, a 100 ml/min flow rate can be measured with an accuracy of $\pm 0.7\%$. This volume flow measurement is called the hydrodynamic flow rate and is the reference flow rate used to estimate the accuracy of ultrasonically determined volume flow rates.

4.1.2 Fluid Flow Relationships

As flow comes out of the upper reservoir the flow velocities are constant across the tube. This type of flow is called plug flow [39]. As the distance along the tube away from the reservoir is increased, the flow velocity profile changes and becomes more parabolic in shape with the maximum flow at the center of the tube and zero flow velocity at the wall of the tube. Beyond a certain length of straight tube, called the entrance length, the flow is fully developed laminar flow. The fluid velocity profile for fully developed steady laminar flow is parabolic and axially symmetric. The required entrance length (E in Figure 9) is given in [39] as

$$E = 0.073 \frac{\rho_0 Q}{\mu} \quad (37)$$

The entrance length for the fluid regulator system is 29 cm (much less than the total length, L) so the maximum fully developed laminar flow rate (determined by solving (28) for Q) is limited to 238 ml/min. Below this rate the actual entrance length is greater than that required, and fully developed laminar flow is established. Above 238 ml/min some degree of plug flow will be present at the ultrasonic measurement point.

The Reynolds number must also be considered. For laminar flow to exist, the Reynolds number must be less than 2000 [39]. Reynolds numbers above 2000 indicate an increased probability of turbulent flow. In very smooth pipes with no external vibration, laminar flow can be established with a Reynolds number as high as 10,000 [39]. The Reynolds number is defined as follows:

$$R = \frac{4 Q \rho_0}{D \mu} \quad (38)$$

For water and the maximum fully developed laminar flow rate of 238 ml/min and a tube diameter of 6 mm, the Reynolds number is 841. This indicates that turbulent flow is very unlikely. At a flow rate of 566 ml/min the Reynolds number is 2000 indicating that turbulent flow may exist in the tube.

4.2 TRANSDUCER POSITIONING SYSTEM

This section describes the mechanical system used to position the transducer such that the ultrasonic beam can be scanned across the dialysis tubing at a fixed measurement angle (the angle between the flow direction and the ultrasonic beam).

The measurement angle can be set for a particular experiment by moving the transducer along a circular track which holds the transducer. The transducer and curved track structure are rotated about the scan axis which is approximately 40 mm away from the center of the dialysis tube and is parallel to the tube. The scan angle is the relative angle about which the track assembly is rotated about the scan axis and determines the position where the ultrasonic beam intersects the dialysis tube. During a volume flow measurement experiment, the scan angle is changed so that the ultrasonic beam samples the two dimensional flow velocity profile inside the dialysis tube.

Figure 10 is a schematic diagram of the angle measurement system employed to determine the scan angle. A Helium-Neon 1 mW laser is directed at a front surface mirror which is mounted so that the scan axis is contained in the plane of the mirror. The transducer is mounted to the scan axis shaft via a curved measurement angle track. The scan axis shaft is held by a long Teflon^R bearing and the mirror is mounted on the top of this shaft. When the transducer is rotated by the scan angle α , as shown in Figure 10, the projection of the laser beam on the meter stick moves an angle 2α . The scan angle, α , is given by

$$\alpha = \frac{1}{2} \arctan\left(\frac{X - 50}{D_M}\right) \quad (39)$$

where

- X = meter stick reading (cm).
 D_M = minimum distance from meter stick to scan axis (cm).

The transducer is rotated about the scan axis by hand, using a spring-loaded micrometer adjustment. Since the meter stick is used to measure the position of the laser beam, the scan angle measurement accuracy is approximately $\pm 0.2\%$ (2 mm out of 1 m). This accuracy can be obtained if the entire length of the meter stick is traversed by the laser beam. The precision of a full scale ($X=100$) scan angle measurement is less than or equal to 0.2% , since the operator should be able to repeatedly read the meter stick within 1 mm standard deviation. For the 6 mm diameter dialysis tube, the ultrasonic beam intersects the tube for a scan angle range of approximately $\pm 4^\circ$. Thus, the accuracy of the scan angle measurement is approximately $\pm 0.016^\circ$ (0.2% of 8°).

The measurement angle (θ) can be changed by sliding the transducer along a specially machined circular track which holds the transducer. Figure 11 shows a cross section of the transducer, dialysis tube and track structure. The radius of the track and the distance from the vessel to the scan axis were designed such that the focus of the transducer remains close to the vessel center. This has been verified by placing a point reflector at the tube center and adjusting the transducer mount for a minimal change in the time of arrival of the reflector's echo as the measurement angle was changed. Typically less than 2% variation in the round trip time of the echo from the point target was obtained for a measurement angle change from 45° to 75° .

4.3 TRANSDUCER ELECTRICAL CHARACTERISTICS

Accurate pulse-echo ultrasound experiments require the transducer to be shock excited with a very short high voltage pulse. This will provide a wideband large ultrasonic pulse and therefore the best axial resolution and energy level. Proper design of the transducer pulser and receiver circuits, described in the next section, requires a detailed knowledge of the electrical characteristics of the ultrasonic transducers to be used.

A simple and practical model of a typical highly damped transducer is shown in Figure 12a. The series RLC circuit represents the acoustic resonance of the piezoelectric material. The electrical resonant frequency of these elements is the same as the acoustic resonant frequency. The resonant frequency for damped, low Q , transducers [40] is primarily determined by the thickness of the piezoelectric material. The quality factor (Q) of the transducer is determined by the properties of the transducer materials. The parallel capacitor (C_0) is determined by the size of the transducer, the dielectric constant of the piezoelectric material and the length of the cable connected to the transducer.

The values of the equivalent electrical elements (R , L , C , C_0) have been determined for each of the three transducers used in this study. The complex impedance of the transducers at many frequencies between 400 kHz and 20 MHz was measured using a vector impedance meter (HP model 4193A). The impedance functions

(magnitude and phase) were then used to calculate the element values for each transducer. At low frequencies (400 kHz) the series RLC circuit has a negligible effect on the overall transducer impedance so that the magnitude of the impedance at 400 kHz was used to determine the value of C_0 . At the resonant frequency of the transducer the RLC circuit is resistive and provides the maximum phase transducer impedance. The quality factor of the RLC circuit was determined by subtracting the admittance of C_0 from the transducer admittance. The resulting admittance function represents the admittance of the series RLC circuit alone. The peak of this admittance curve is the admittance of R. The two frequencies at which the admittance falls -3dB (0.707) below the peak admittance was used to determine the bandwidth of the transducer. The quality factor and the RLC elements were then determined.

The values of the transducer model elements for each of the three focused transducers (Panametrics models V307, V309, and V3323) are shown in Figures 12b, 12c and 12d, respectively. The transducers will be referred to by number, as indicated in Figure 12. Note that the 10 MHz transducer (transducer number 3) has an inductor inside the body of the transducer in order to improve the electrical matching when long cables are used. All three transducers have a two inch focal length. Table 3 summarizes the characteristics of each transducer.

Table 3 - Characteristics of Three Panametrics Transducers.

Transducer Number	Aperture Size (mm)	Center Frequency (MHz)	Quality Factor	Capacitance at 400 kHz (pF)
1	25.4	5.0	2.0	1250
2	12.7	5.0	2.8	1120
3	25.4	10.0	2.8	330

4.4 TRANSDUCER PULSER-RECEIVER

The circuit used to shock excite and receive the ultrasound echoes from the transducers described above is shown schematically in Figure 13. A vertical field effect transistor (VFET, RFP15N15) is used to develop a large current, I_0 , in an inductor, L_1 . A driver integrated circuit (DS0026) is used to turn the VFET off when the trigger signal from the ultrasound data acquisition system (UDAS, see section 4.5) goes high. When the magnetic field established in L_1 collapses, the voltage on the drain of the VFET is given by the following equation:

$$V = I_0 \sqrt{\frac{L_1}{C_t}} \sin \frac{t}{\sqrt{L_1 C_t}}, \quad 0 \leq t \leq \pi \sqrt{L_1 C_t} \quad (40)$$

where C_t is the total capacitance of the VFET, transducer and diode circuits. After the half cycle of the sinewave is completed, D_1 conducts and the VFET is turned back on when the trigger pulse returns to zero. During the transmit phase just described, D_2 is conducting so that the voltage on the transducer is almost the same as the voltage on the drain of the VFET.

After the VFET is on and the transducer voltage is near zero, D2 opens and any voltage appearing on the transducer appears across the wideband toroidal transformer, T1. The impedance of the transducer (20-30 ohms at center frequency) is transformed to 50 ohms by T1 for easy transmission to the wideband amplifiers located several meters away. This transformation also increases the signal level without increasing the relative noise level. The inductance of the primary winding of T1 also insures that D2 turns off (after about 20 μ S) causing the pulser-receiver to switch from transmit to receive mode.

The received signal from transformer T1 is fed into an amplifier (Aydin Vector MHD-175) with a gain of 50 dB and a bandwidth of 40 MHz. The amplified echoes are then directed to the UDAS for further signal processing.

Figure 14 shows the result of a SPICE simulation [41] of the pulser output and acoustic pressure output for each of the three transducers. The measured values of the transducers shown in Figure 12 were used in each case. Each of the simulated results have two waveforms superimposed on each plot. The shorter waveform in each plot is the simulated pulser output in volts at the transducer terminals. In each case the peak voltage is limited to 150 V, which is the breakdown voltage of the VFET. A different initial inductor current (I_0) for each transducer is indicated for each case. This current is adjusted by changing the VFET power supply voltage. The longer duration waveform is the voltage across the radiation resistor (R in Figure 12a) which is proportional to the acoustic pressure emitted by the

transducer. It can be seen from Figure 14 that the acoustic pressure waveform is primarily determined by the transducer acoustic quality factor. Because the pressure waveform is longer than the excitation, the bandwidth of the excitation is larger than the pressure waveform and does not significantly affect the bandwidth of the transmitted waveform.

4.5 ULTRASOUND DATA ACQUISITION SYSTEM

The Ultrasound Data Acquisition System (UDAS) provides the basic requirements to collect and analyse ultrasound echoes in the frequency range between 0.5 MHz and 15 MHz [42]. An overall block diagram of the UDAS is shown in Figure 15. A 1 V peak-to-peak signal can be sampled every 20 ns with the 8 bit A/D converter. The unit provides a periodic short pulse to trigger an ultrasound pulse generator at a rate up to 30 kHz. At a programmable time after the trigger, a number of samples are stored in the high speed memory. This programmable range gate can be repeated a given number of times as requested. Up to 393,216 (384 Kbytes) total samples can be stored in the data memory.

A Z-80 microcomputer controls the operation of the UDAS. This computer has a serial interface for communication with a larger host computer, a DMA controller for data memory transfers, an external data bus interface for communication with other devices and an ASCII keyboard interface for data entry. All of these circuits are contained on the UDAS CPU board and control the remaining circuit boards.

The A/D converter interface board contains a TRW TDC1026E1C 50 MHz 8 bit A/D converter, high speed data registers, and clock generation circuits. The 8 bit ECL output of the A/D converter is stored in eight, 4 bit shift registers every 20 ns. At the end of four clock cycles, the four data bytes are transferred to a 32 bit register. The output of this register appears on the 32 bit high speed DMA bus for storage in the memory. This technique reduces the bandwidth requirement of the DMA bus by a factor of 4.

Data appear on the 32 bit DMA bus every 80 ns. In order to store these data, memory with a cycle time of 80 ns or less would be required. This type of memory is very expensive and has a low density (bits/package). For these reasons, slower NMOS static memory was chosen for this application. To allow these slower memories to be used, data on the 32 bit DMA bus are latched into 4 different memory boards in a sequential fashion.

Since a large memory size is required for many ultrasound experiments, each memory board was designed to contain 32 Kbytes of data memory. The cycle time of each individual memory board is 320 ns. Since each dual port memory board is addressed as one of four consecutive bytes, a minimum of four memory boards are required in order to have continuous memory. In the current system, 12 memory boards were fabricated for a total of 384 Kbytes. Unlike the other components of the system, the memory boards are printed circuit boards to reduce the cost of construction.

The dual port memory boards can be accessed in one of two ways. As described above, data from the A/D converter DMA board are transferred into the memory via the 32 bit DMA bus. The Z-80 microcomputer can also read and write any location in the data memory via the 8 bit bidirectional (tri-state) data bus. This allows for analysis as well as storage of the data. The high speed correlator board also uses this 8 bit bus to fetch the values to be correlated from data memory.

The display board contains two types of video display devices. The alphanumeric portion provides 80 columns by 24 lines of characters with cursor addressing and other standard terminal functions. The graphic portion of the display board provides a computer addressable 256 x 192 pixel black and white display mode or a lower resolution color display mode. These two commercially available boards are mounted together on the UDAS display board.

In order to analyze the ultrasound echoes, a high speed correlator board was designed. This board acts as a DMA device and accesses the data memory directly at a 2 Mbyte/sec rate. Every microsecond, two samples can be multiplied and added to the previous result. Thus, one million multiply-accumulate operations are performed each second. The DMA address generator on the multiplier-accumulator board provides two addresses and can sequence through up to 256 locations. Thus the high speed correlator board performs the function

$$C = \sum_{i=0}^{N-1} X(a + i) Y(b + i) \quad (41)$$

where

X(a) = first data value fetched (8 bits: -128 to 127)
 Y(b) = second data value fetched (8 bits: -128 to 127)
 a = start address of first data
 b = start address of second data
 i = address sequence value
 N = number of values to accumulate (1 to 256)
 C = final result

This function is a discrete correlation and can be used to perform a variety of signal processing tasks [43]. Absolute time of flight can be estimated by searching for the maximum correlation of the echo data with a stored waveform. A short length DFT can be implemented by precalculating the DFT coefficients and storing them in memory. Digital filtering or convolution can also be accomplished in a straightforward manner. All of the preceding tasks would require excessive computation on a host computer, especially if the data set size were large. Furthermore, the time required to send the data to another computer over serial communication lines may be orders of magnitude greater than the computation time using the multiplier accumulator. Other tasks, such as floating point operations, can be accomplished much easier on a host machine.

4.6 CALIBRATION OF MEASUREMENT SYSTEM

Any ultrasonic measurement of distance is dependent on two factors: the speed of sound of the medium and the measurement of the time delay. One uncertainty in the measurement of delay is

the delay introduced by the transducer, pulser-receiver and UDAS. Since this delay is difficult to estimate, it has been measured experimentally as described in the next section. Section 4.6.2 describes a time of flight method used to measure the speed of sound of the materials used in this study.

4.6.1 Range Estimate Accuracy

The time delay from the UDAS trigger pulse to the time an echo signal is sampled by the A/D converter is determined by counting the number of 50 MHz clock periods with a simple counter circuit. The counter value at the time the range gate starts indicates the number of 20 ns periods which have occurred since the trigger. The time of arrival of the ultrasonic echo reflected from a target is dependent on the speed of sound in the medium, the speed of sound of the transducer material, the delay introduced by the pulser-receiver electronics and the length of the cables between the UDAS and the pulser-receiver. Since some of these delay elements do not change from one experiment to the next, they can be lumped together into a single constant, t_0 . The time of arrival of a hypothetical target at the front face of the transducer is t_0 . Thus, the datum or reference of the range measurement is the front face of the transducer.

Figure 16 shows the results of a known position versus time of arrival experiment. A mill base (which is calibrated in inches) was used to accurately position a 1.4 mm diameter stainless steel pointed target. The point target was first positioned at the front face of the transducer in the center of

the transducer aperture. The target was then moved away from the transducer a distance of 1.600 inches (40.64 mm). The echo was sampled by the UDAS and sent to the host computer for later processing. The target was moved 0.01 inches (0.254 mm) away from the transducer and this echo was sampled and sent to the host computer. This process was repeated until the signal was reduced to less than a tenth of the peak amplitude near the focus at approximately 1.92 inches (48.768 mm). The signal level at the UDAS input was adjusted so that the peak echo amplitude was within 10 A/D converter levels of the maximum clipping level (127 or -128). The time of the first transition away from the average signal level of the echo was used to estimate the time of arrival of the echo. In order to be considered as the arrival time, the transition away from the average must be greater than three quantization levels and must be followed by a large peak (greater than 50 quantization levels away from the average). Since the signal-to-noise ratio is very large (greater than 40 dB) this simple technique can be used to determine the time of the first ultrasonic response due to the target. Figure 16 is a plot of the resulting time of arrival of the echo versus distance from the transducer face and will be used to estimate the absolute distance at each range cell. This result, shown for transducer number 1 at 25°C, gives an approximate speed of sound (1.496 mm/ μ s) and the time from the trigger pulse for a hypothetical target at the transducer face ($t_0 = 3.331 \mu$ s). Both of these estimates are determined from a linear least squares fit of the mill base experimental data. The t_0 estimate is the time axis intercept of the best fit line from the 51 data points, and

the speed of sound is the inverse of the slope estimate. The standard deviation of the time of arrival estimate obtained from the regression is $0.0233 \mu\text{s}$. The mill base positioning system is accurate to $\pm 0.04 \text{ mm}$ ($\pm 0.001 \text{ inches}$). Thus, the 95% confidence interval of the least squares zero time of arrival estimate is $\pm 0.0823 \mu\text{s}$ [45]. The estimate of the range distance from the face of the transducer to the center of a particular range cell is given by

$$R = c_0 (t/2 + t_g/4 - t_0) \quad (42)$$

where

- R = range from face of transducer to the range cell (mm).
- t = start time of range gate (μs).
- t_g = range gate length (μs).
- t_0 = time of arrival at front face (μs).
- c_0 = speed of sound in medium (mm/ μs).

4.6.2 Speed Of Sound Measurement

The speed of sound was measured for each of the fluids used in this study. The time of flight of a 5 MHz ultrasonic pulse (about five cycles) was measured for distilled water, normal saline, a 50% glycerin-water mixture, and porcine blood. Distilled water and normal saline were used as a surrounding fluid (speed of sound c_0). In all cases, the temperature of the 40 liters of surrounding fluid was held at a constant temperature of 25°C ($\pm 0.5^\circ\text{C}$) by the temperature controlled water bath (EX-500). Distilled water, a 50% glycerin-water mixture and porcine blood were used as scattering fluids (speed of sound c_1) inside the tube. The 50% glycerin-water mixture was made by mixing 500 ml of glycerin with 500 ml of distilled water.

Water with Sephadex^R particles and the 50% glycerin-water mixture with Sephadex^R (Sephadex^R G-50, particle size 20-70 μm) were measured. Sephadex^R was added to distilled water and the glycerin-water mixture in order to scatter ultrasound in a similar way that red blood cells scatter ultrasound. Approximately 2 grams of Sephadex^R per liter of fluid (either water or glycerin) were added in each case. This amount of Sephadex^R resulted in strong ultrasonic backscattering (greater than 20 dB signal to noise ratios) and, additionally, remained suspended in the fluid. Approximately one liter of porcine blood was collected into 200 ml of anti-coagulant solution, and this mixture was measured within one hour. The anti-coagulant solution was 2.2% sodium citrate and 2.5% dextrose.

Each speed of sound measurement proceeded as follows. A small Plexiglas^R tube (0.75 inch inner diameter by 1.00 inches long) with a transducer mounted on one end of the tube was used to hold the fluid to be measured. The transducer was an unfocused transducer with a 0.75 inch (1.91 cm) diameter aperture and a resonant frequency of 5 MHz (Panametrics model number A308). The fluid to be measured was poured into the open end of the inverted Plexiglas^R tube on top of the front face of the transducer. The transducer and Plexiglas^R tube assembly was mounted so that gravity makes the top surface of the fluid perpendicular to the tube wall and the ultrasonic beam. The UDAS and pulser receiver were used to record the first two echoes caused by reflection of the ultrasound by the top surface of the fluid being measured. The first echo occurs at the time required

for the ultrasound to travel from the transducer to the fluid surface and back. The second echo occurs at the time the first echo reflects off the front of the transducer, travels to the fluid surface, and back to the transducer. As indicated in [44] the time between the first and second echoes is a good indication of the round trip time required for the ultrasonic pulse to travel twice the length of the fluid being measured. A total of 2048 samples were stored by the UDAS (representing $40.96 \mu\text{s}$) and then sent to the host computer where the time between the first two echoes was estimated. This round trip time was estimated by locating the time shift between the echoes which gave a maximum correlation. This time shift was estimated to the nearest sample period ($0.02 \mu\text{s}$) providing a very accurate ($\pm 0.07\%$ of $35 \mu\text{s}$) estimate of the time between echoes.

The uncertainty in this speed of sound measurement can be traced to the ability of the experimenter to maintain the fluid level of each liquid equal to the total length of the Plexiglas^R tube. The length of the Plexiglas^R was measured to be 1.035 inches (26.29 mm) with an accuracy of $\pm 0.01\%$. The round trip length is therefore 52.58 mm. The measurement of the time of flight by the correlation method is accurate to better than $\pm 0.02 \mu\text{s}$ (one sample period). Thus, the uncertainties of the distance measurements and the time of flight measurements are small compared to the uncertainty in setting the column height to the length of the tube.

By doing a series of time-of-flight experiments with water, the fluid level uncertainty appears to be about $\pm 0.9\%$. Thus, the speed of sound results shown in Table 4 reflect this uncertainty:

Table 4 - Speed of Sound of Materials Used in This Study.

Material	Temperature ($^{\circ}\text{C}$)	Round Trip Time (μs)	Speed of Sound ($\text{mm}/\mu\text{s}$)
Water	24.8	35.1	$1.50 \pm .01$
Normal Saline	25.0	34.7	$1.51 \pm .01$
Porcine Blood	24.9	33.9	$1.55 \pm .01$
50% Glycerin	24.4	30.9	$1.70 \pm .02$

The values for water, saline, and 50% glycerin agree with published values [44]. The speed of sound measurement obtained from the slope of the mill base experimental data ($1.496 \text{ mm}/\mu\text{s}$, see the previous section) also agrees with the measured value for distilled water. The values in the literature for various kinds of blood at 25°C vary from 1.52 to $1.58 \text{ mm}/\mu\text{s}$ depending on the species and type of anti-coagulant used [27,28,46,47].

In order to determine the change in the speed of sound due to adding Sephadex^R particles to the distilled water and the glycerin-water mixture, two experiments were performed. Both experiments measured the change in the speed of sound caused by the addition of the Sephadex^R to one of the fluids. Since the column height remains virtually unchanged by adding the Sephadex^R, the change in the time of flight can be used as a good indicator of a change in the speed of sound. With the fluid in the measurement tube (volume 8 ml), 0.16 grams of Sephadex^R was

added (2 grams/l). The round trip time before and after the Sephadex^R was added was then compared. The round trip time of the distilled water changed less than 0.5% when the Sephadex^R was added. Sephadex^R was added to the glycerin-water mixture resulting in less than 0.5% change in the round trip time. Thus, the speed of sound of each fluid is assumed to be the same with Sephadex^R as without Sephadex^R.

CHAPTER 5

AXIAL FLOW VELOCITY MEASUREMENT METHOD

This chapter introduces the methods used to measure the axial flow velocity by the time domain correlation technique. The axial flow velocity is the component of the flow velocity in the direction of the ultrasound beam axis. All ultrasound measurements of motion determine the velocity of the scatterers in the direction of propagation of the ultrasonic energy [10,12]. Thus, the flow velocity must be calculated from the axial flow velocity measurement and the angle between the axis and the direction of flow (measurement angle, θ , as defined in section 3.1). In Chapter 6, methods to estimate the measurement angle from a collection of axial velocity measurements will be discussed; in the current chapter methods to measure the axial flow velocity versus range are described.

The axial flow velocity can be determined at different positions within the vessel, at different times, or both. In this thesis, axial flow velocity measurements versus time are defined as pulsatile flow measurements independent of the type of flow being measured (steady or pulsatile). The pulsatile flow measurement method used in this research will be described in section 5.3. One dimensional flow measurement is defined as the

measurement of the axial flow velocity versus range along the ultrasonic beam axis. The previous research [12] was limited to one dimensional flow velocity measurements. Two dimensional flow measurements are determined from the axial flow velocity measurements versus range along the beam as a function of scan angle. The two dimensional flow measurement method, described in the next chapter, will be used to determine the measurement angle (θ) and the volume flow. This section provides the details of the axial flow velocity measurement method developed by using the ideas presented in Chapter 3 and [12].

The theoretical precision of the time domain correlation method derived in Chapter 3 and illustrated in Figure 8 indicates that significant changes in the precision of the axial flow measurement occur as the mean time shift or measurement angle is changed. The precision value for each measurement angle also depends on the signal-to-noise ratio. The signal-to-noise ratio only scales the precision curve; the shape of the curve remains the same. Because of this scaling property, the relative variance curve (variance divided by the minimum variance throughout all time shifts) of a flow estimate for a particular transducer and particular measurement angle can be determined a priori.

Since the relative variance curve does change with measurement angle, the location of the minimum variance cannot be determined unless the measurement angle is known. By using a series of flow measurements of constant flow the variance of the flow estimates can be determined for the particular transducer

configuration. The estimate of the variance by this method assumes that the flow remains constant throughout the measurement interval. This is a reasonable assumption when steady flow is measured, but clearly not valid when pulsatile flow is measured. For this reason, pulsatile flow measurements (to be considered in more detail later) require prior knowledge of the measurement angle.

The next section describes the method used to determine a flow estimate of steady flow so that the minimum variance is obtained throughout all range positions. Section 5.2 describes the programs used to determine the time shift of the echoes by discrete time domain correlation.

5.1 MINIMUM VARIANCE FLOW

If only two echoes are used to measure the axial velocities at many range positions, the precision of the measurement at the midstream position can be made small (by proper choice of the time between echoes) but at ranges near the vessel walls, the shift will decrease and the imprecision of the flow estimate will increase dramatically. This increase in the imprecision was illustrated in Table 2 obtained from [12].

In order to provide a flow estimate with the minimum variance throughout all range positions, a series of N consecutive echoes each separated by the pulse repetition period, T , can be used effectively. At a particular range the axial flow velocity estimate for a pair of echoes is given by

$$V_{AE} = \frac{c S(i,j)}{2 (j - i) T} \quad (43)$$

where

- V_{AE} = magnitude of the axial flow velocity of the scatterers at a particular range estimated from two echoes.
 $S(i,j)$ = time shift between the i th and j th echo ($j > i$).
 T = pulse repetition period.
 c = speed of sound in the scattering fluid.

By averaging these estimates together in an optimum weighted fashion, a minimum variance axial flow velocity estimate \hat{V}_A can be determined (method to be described below). From this optimum axial flow velocity value, the magnitude of the flow velocity in the tube, \hat{V} , is

$$\hat{V} = \hat{V}_A / \cos(\theta_M) \quad (44)$$

where θ_M is the estimate of the angle between the ultrasound beam axis and the velocity vector (measurement angle).

The relationship between \hat{V}_A and V_{AE} is determined by the method used to average the V_{AE} values. One solution for \hat{V}_A is simply to average all the V_{AE} estimates with equal weight applied to each. This is not practical because the precision of each V_{AE} value varies as a function of position and time shift. For example, if the average shift between consecutive echoes ($j-i=1$) were 50 ns, then the average shift between every other echo ($j-i=2$) would be 100 ns. The precision of the shift estimate with a 100 ns mean shift would be considerably smaller in the typical case illustrated in Figure 8.

As shown in [10,12,34] the backscattered ultrasound signal from a scattering fluid is approximately Gaussian and each echo is an independent measurement. Since new scatterers with different random orientations and positions are constantly entering the ultrasonic beam, the echoes should be independent. As was assumed in the theory and by other results [12], the axial flow velocity estimates, V_{AE} , are assumed to be independent Gaussian random variables which can be completely specified by a variance and a mean.

Since each V_{AE} estimate is assumed to be an independent measurement, all pairs of echoes can be used to determine a shift value and corresponding velocity value at each range position. There are N consecutive echoes resulting in $N-1$ pairs of consecutive echoes ($j-i=1$) which can be used to determine $N-1$ V_{AE} estimates of the axial flow velocity. Also, there are $N-2$ pairs of echoes separated by $2T$ (using every other echo), $N-3$ pairs of echoes separated by $3T$ (using every third echo), and so on. Thus, at each range position, a total of $N(N+1)/2$ axial velocity values (V_{AE}) can be determined from N echoes. Because the mean time shift for each $S(i,j)$ changes with the number of periods between echoes, some of the axial velocity estimates will have much larger variances than other V_{AE} values. It is, therefore, desirable to weight axial flow velocity estimates with a small variance more heavily than other axial flow velocity values with a larger variance. Let $W(i,j)$ be a set of weighting coefficients. The axial flow velocity estimate is given by

$$\hat{V}_A = \frac{c}{2T} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{W(i,j) S(i,j)}{j-i} \quad (45)$$

$$\text{where } \sum_{i=1}^{N-1} \sum_{j=i+1}^N W(i,j) = 1 \quad (46)$$

The variance of this estimate is

$$\text{Var}[\hat{V}_A] = \frac{c^2}{4T^2} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{W(i,j)^2}{(j-i)^2} \text{Var}[S(i,j)] \quad (47)$$

or,

$$\hat{V}_A = \sum_{i=1}^{N-1} \sum_{j=i+1}^N W(i,j) V_{AE}(i,j) \quad (48)$$

$$\text{Var}[\hat{V}_A] = \sum_{i=1}^{N-1} \sum_{j=i+1}^N W(i,j)^2 \text{Var}[V_{AE}(i,j)] \quad (49)$$

Since all the echoes are assumed to be independent samples of Gaussian random processes [10,12], the flow estimates obtained from steady flow are stationary to order 2 since during the N echoes the mean and the variance of the process will not change. This assumption greatly simplifies the optimum axial velocity estimate for steady flow. Therefore, the variance of the shift estimate does not depend on the time the echoes are sampled. Thus,

$$\text{Var}[S(i,j)] = \text{Var}[S(j-i)] = \text{Var}[S(\text{DEL})] \quad (50)$$

where DEL is the number of time periods, $j-i$, between echoes. With the above assumption, the optimum axial flow estimate is as follows:

$$\hat{V}_A = \frac{c}{2T} \sum_{\text{DEL}=1}^{\text{DELMAX}} W(\text{DEL}) \left[\frac{1}{N - \text{DEL}} \sum_{i=1}^{N-\text{DEL}} \frac{S(i, i+\text{DEL})}{\text{DEL}} \right] \quad (51)$$

Because $\text{Var}[S(\text{DEL})]$ depends only on the time shift between echoes, the variance of the velocity estimate can be minimized by proper selection of the weighting coefficients ($W(i,j)$). The variance function can be determined a priori for a particular transducer configuration. With this a priori information and the $S(i,j)$ values, equations (45) and (46) can be used to solve for the optimum set of $W(\text{DEL})$ values. The solution for independent Gaussian variables, given in [48], shows that the weighting coefficient is inversely proportional to the variance of the estimate. This can be expressed as

$$W(\text{DEL}) = \frac{K \text{ DEL}^2}{\text{Var}[S(\text{DEL})]} \quad (52)$$

The value of the constant, K , can be determined by combining equation (37) and (41), giving

$$W(\text{DEL}) = \frac{\text{DEL}^2 (\text{Var}[S(\text{DEL})])^{-1}}{\sum_{k=1}^{\text{DELMAX}} k^2 (\text{Var}[S(k)])^{-1}} \quad (53)$$

The denominator of equation (53) is simply the sum of the variances of the V_{AE} estimates and assures that the sum of the weighting function is unity.

For a large number of echoes ($N > 30$) a reasonable estimate of the $\text{Var}[S(\text{DEL})]$ can be obtained from the shift estimates by the discrete summation

$$\text{Var}[S(\text{DEL})] = \frac{1}{N-\text{DEL}} \sum_{i=1}^{N-\text{DEL}} S(i, i+\text{DEL})^2 - \left[\frac{1}{N-\text{DEL}} \sum_{i=1}^{N-\text{DEL}} S(i, i+\text{DEL}) \right]^2 \quad (54)$$

where the second term is the average shift squared for each DEL value. The $\text{Var}[S(\text{DEL})]$ and the $W(\text{DEL})$ weighting function can be calculated for $1 \leq \text{DEL} \leq \text{DELMAX}$ where DELMAX is the maximum $j-i$ value considered. $W(1)$ and $W(\text{DELMAX})$ should both be small compared to the maximum weighting function for best performance. Thus, an optimum axial velocity of a scatterer at each range along the vessel can be obtained. The variance of the weighted velocity estimate is found by substituting (53) into (47). The result is

$$\text{Var}[\hat{V}_A] = \frac{c^2}{4 T^2 \sum_{\text{DEL}=1}^{\text{DELMAX}} \frac{\text{DEL}^2}{\text{Var}[S(\text{DEL})]}} \quad (55)$$

The $\text{Var}[\hat{V}_A]$ is always less than or equal to the smallest $\text{Var}[S(\text{DEL})]/\text{DEL}^2$ term, as can be seen from equation (55). If the $\text{Var}[S(\text{DEL})]/\text{DEL}^2$ terms were equal for all DEL, then by equation (55),

$$\text{Var}[\hat{V}_A] = c^2 \text{Var}[S(1)] / 4 T^2 \text{DELMAX} \quad (56)$$

Equation (56) represents a significant improvement in the variance compared to the variance associated with estimating the axial velocity from only one DEL value ($c^2\text{Var}[S(1)]/4 T^2$). Since all of the individual variances ($\text{Var}[S(\text{DEL})]/\text{DEL}^2$) are not equal, the improvement in the variance of the optimum axial velocity estimate compared to the smallest variance of the individual variances at each DEL value is not as large as predicted by (56). The improvement in the variance of the optimum axial velocity estimate can be expressed as the ratio of the minimum variance of the individual estimates for all DEL values to the variance of the optimum estimate given by (55). This ratio is always greater than one and is expressed by

$$\frac{\text{Min}(\text{Var}[V_{AE}])}{\text{Var}[\hat{V}_A]} = \text{Min} \left(\frac{\text{Var}[S(\text{DEL})]}{\text{DEL}^2} \right) \sum_{\text{DEL}=1}^{\text{DELMAX}} \frac{\text{DEL}^2}{\text{Var}[S(\text{DEL})]} \quad (57)$$

Where the $\text{Min}(\)$ operator indicates the minimum of all DEL values ($1 \leq \text{DEL} \leq \text{DELMAX}$).

5.2 DISCRETE MAXIMUM CORRELATION METHOD

The time shift between echoes is estimated by determining the time shift with the maximum correlation between two echoes, as indicated in Chapter 3. Unfortunately, the correlation function has many local maximum values (see Figure 6) due to the bandlimited nature of the echoes, and the correlation between echoes can only be estimated at discrete multiples of the sampling rate. This section provides the details of programs used to estimate the continuous time shift between echoes and the methods used to avoid the problems associated with the multiple peaks of the correlation function.

With a UDAS sampling rate of 50 MHz, the correlation function of each pair of echoes can be calculated at discrete shifts of 20 ns each. As shown in the theoretical precision versus time shift curves (Figure 8), the velocity estimate with the best precision is obtained with a 100-200 ns time shift. Thus, for best precision, the pulse repetition period should be set so that the shift with maximum correlation is between five and ten samples. Suppose that the correlation between echoes is determined at ten discrete values from zero to nine samples. The shift with the largest correlation is used as the shift estimate, $S(i,j)$, as in equation (43). The shift estimates are then averaged in an optimum fashion as described in the previous section. The variance due to the discrete nature of the shift estimate would significantly increase the variance of the flow estimate.

For illustration purposes, the error introduced by the discrete estimate of the time shift can be modeled as a zero mean uniformly distributed random variable from -0.5 to 0.5 samples as is often done in the analysis of quantization of analog signals [43]. The variance of this error is 0.084 (variance of the uniform error distribution). The precision of a flow estimate would be limited to 5.8% by this additional error if the continuous case had a zero variance.

For example, if the mean time shift to be estimated were five samples, then the precision of a continuous correlation estimate would be less than 5% for signal-to-noise ratios greater than 20 dB, a 45° measurement angle, and an RMS bandwidth of 2.5 MHz (see Figure 8). The variance due to the discrete correlation and the variance of the continuous time correlation estimate can be added together by making the assumption that the two errors are independent. This is a reasonable assumption used in almost all quantization problems in digital signal processing. The sum of the variances of a continuous time correlation flow estimate with a 5% precision (0.061 variance) and the discrete error (0.084 variance) is 0.145 . The precision of this discrete time flow estimate is, therefore, approximately 7.6% . This considerable increase in the standard deviation of the flow estimate indicates the importance of computational methods which estimate the shift with the maximum correlation more precisely by interpolation of the discrete correlation values.

Techniques which estimate the peak correlation of two signals by iterative methods are discussed in [49] and [50]. These techniques require a great deal of computation, since a large portion of the correlation function must be calculated. Frequency domain methods are also indicated in [50]. Since these methods require that a number of FFTs be computed, the computation time is again excessive on a general purpose computer. For these reasons, the peak of the correlation function was estimated by fitting a parabolic curve to the peak correlation point and the two nearest neighbors. This technique was used by Foster [12] and analysed extensively by Boucher and Hassab [51].

The variance and bias due to the parabolic fit of the correlation peak were determined theoretically for a bandpass signal. The results in [51] show that the variance of the time shift estimate from the parabolic-fit peak method was slightly larger than the variance from methods which calculate the peak by band-limited interpolation of the correlation function [49]. The bias of the parabolic-fit peak method was also derived in [51]. The time shift estimate was shown to be unbiased when the peak was at a discrete number of samples or half way between the two sample points. The bias of the shift estimate between these unbiased values changes with the signal-to-noise ratio and signal bandwidth. Independent of the noise level, the bias increases to a maximum value at one quarter and three quarters of the time between samples.

The example given in [51] was for a 0 dB signal-to-noise ratio and a bandwidth of 30% of the sampling rate. This worst case example showed an estimated shift of 0.22 when the actual shift was 0.25. If the shift to be estimated was 5.25 samples, then the discrete correlation method would give a 5.22 sample result. This represents a -0.6% error in the shift estimate. For larger shifts or greater sampling rates this inaccuracy becomes smaller.

Figure 17 shows a flow chart of the maximum correlation algorithm. The algorithm works properly when the pulse repetition rate is large enough so that the maximum correlation for two adjacent echoes ($DEL=1$) occurs at less than five samples. The first set of correlations is calculated at $-2, -1, 0, 1, 2, 3, 4,$ and 5 samples of shift between echoes. The algorithm first finds the local maximum of the discrete shifts. If the correlation between the echoes for a particular shift is negative, the next correlation at the next shift value is calculated. Once a positive correlation between the echoes is found, the correlations for the next two shifts are determined. If the second correlation ($PREV1$ in the flow chart) is not greater than both the first positive correlation ($PREV2$) and the third correlation (ACC , the most recent value) then additional correlations are performed until a local maximum is located. Once the maximum $PREV1$ value is located, no more correlations need be calculated to estimate the shift. The fractional portion of the shift is then calculated from the three correlation values by the parabolic-fit method (see equation shown in Figure 17).

If the shift of the echoes is at a point greater than five samples, the multiple peaks in the correlation function could cause an erroneous estimate of the maximum correlation shift. The pulse repetition period must be set so that the peak shift between consecutive echoes is between one and four samples. If the shift is less than one sample, the variance of the flow estimate for DEL=1 will be very large and will not significantly contribute to the improvement of the optimum velocity estimate indicated by equation (54). In other words, the variance of the DEL=1 term would be so large compared to the other variances that little can be gained by calculation of the shifts.

Then, every other echo (DEL=2) is used to estimate the shift by starting at two samples less than twice the shift estimate for adjacent echoes. Thus, the shift of the next DEL value is guessed from the shift of the previous DEL value. As shown in Figure 17, the starting point for the next shift estimate, GUESS, is

$$\text{GUESS} = E[S(\text{DEL})] (\text{DEL} + 1) / \text{DEL} \quad (58)$$

where $E[S(\text{DEL})]$ denotes the mean shift calculated for the previous DEL value. This technique is used to estimate the shift from DEL=1 to DEL=10. The optimum flow estimate is then made from these ten average shift estimates as discussed in section 5.1.

The calculation of all of the discrete correlations was done on the UDAS using the high speed correlator. The floating point calculations such as the variance of the flow estimate and the optimum flow rate estimate were done on the host computer, a VAX

11/730. About 90 seconds was required to perform the correlation calculations for 384 echoes at 25 range positions and to send the average shift data to the host computer using the UDAS.

5.3 PULSATILE FLOW MEASUREMENT METHOD

The measurement of pulsatile flow is considerably more difficult than the measurement of steady flow. Pulsatile flow is flow with a periodically varying flow rate [39]. Variation of the flow with time makes the flow estimates nonstationary, since the mean of the flow estimate is time dependent and since the variance of the shift estimates is not independent of the pair of echoes used to estimate the flow, as assumed in equation (39). However, by assuming that over a short time interval the flow does not change significantly, the flow estimates can be assumed to be stationary. Future research where pulsatile flow measurement is considered in more detail may address these assumptions and resulting problems in greater depth.

The shifts can be calculated at each DEL value from DEL=1 to DEL=10 by the algorithm illustrated in Figure 17. For pulse flow measurements, these values of shift are not averaged together but are simply sent to the host computer for further processing.

If the measurement angle and weighting function for a particular measurement are known, the optimum flow estimate can be determined. Since the flow estimates are not stationary, the estimate of the variance from the flow data as was done for steady flow is not possible. With the a priori knowledge of the

weighting function determined for steady flow at the same measurement angle, the optimum flow estimate, \hat{V}_A , can be determined in exactly the same way as indicated by equation (42), except that the number of echoes used must be much smaller than the number used for steady flow measurements (384 using the UDAS).

If the pulse repetition rate is 5000 Hz, ten echoes represent 2 ms of time. For typical human blood flows, the pulse variation rate is never greater than 3 Hz (the human pulse rate rarely exceeds 180 beats/min). Assuming that the magnitude of the frequency components in the pulsatile flow waveform are zero above the tenth harmonic, the flow will change less than 1% during the 2 ms represented by ten echoes. Since the precision of a typical measurement is 5%, this additional 1% variation due to the time variation of the flow during the ten echoes is not significant.

The pulsatile flow measurement technique uses ten echoes for each optimum flow estimate ($N=10$ in equation (42)). Thus, a total of 55 shift estimates ($N(N+1)/2$) are used to estimate each optimum axial flow velocity by equation (42). The $W(\text{DEL})$ values are determined from measurements of the variance versus shift ($\text{Var}[S(\text{DEL})]$) for steady flow at the same measurement angle and transducer. Further discussion of the validity of this method will be presented in Chapter 7.

CHAPTER 6

VOLUME FLOW MEASUREMENT METHOD

The measurement methods used to estimate volume flow from a collection of axial flow velocity measurements are considered in this chapter. In particular, the axial flow velocity estimates determined from consecutive echoes by time domain correlation will be used to calculate the steady flow in a round tube. The steady flow velocity of fully developed laminar flow is parallel to the vessel walls and is parabolic in shape with a peak flow velocity at the center of the vessel. For any viscous fluid, the flow at the walls must be zero [39]. Since steady, fully developed laminar flow is one of the simplest types of flow to measure and describe, it will be considered exclusively in the volume flow measurement method presented here.

A two dimensional set of constant axial velocity points is determined from a series of one dimensional axial flow velocity profiles as described in section 6.1. An elliptic fitting algorithm used to determine the position of constant axial flow velocity contours from the constant velocity points is described in section 6.2. Two different methods to estimate the volume flow from the constant velocity contours are presented in section 6.3.

The first method, called the INTEGRATED volume flow estimate, determines the size of the tube and the measurement angle from ten constant velocity ellipses. The size of the constant velocity ellipses is estimated from the velocity versus range data. The volume flow contributions from each elliptically shaped section are integrated to yield the INTEGRATED volume flow estimate.

The second volume flow estimate, called the TUBE SIZE volume flow estimate, determines the best fit ellipse of the zero velocity points. The size of this tube wall ellipse and the maximum axial velocity measurement (assumed to be at the center of the tube) are then used to form the volume flow estimate. This estimate assumes that the flow profile is parabolic and axially symmetric, that is, fully developed laminar flow.

In the remainder of this chapter, the effects of refraction on the accuracy of the volume flow estimates are described. A method to correct experimental axial flow velocity measurements for the effects of refraction is discussed in section 6.4. The results of volume flow measurement simulations used to establish the accuracy of the volume flow measurement methods are presented in section 6.5. The results of several simulations which quantify the effects of refraction on the volume flow estimates are presented in section 6.6.

6.1 TWO DIMENSIONAL CONSTANT VELOCITY POSITIONS

The method used to estimate the two dimensional axial flow velocity positions from the one dimensional axial flow velocity profiles is described in this section. The characteristics of a typical axial flow velocity profile for fully developed laminar flow in a unit radius tube is described in section 6.1.1. The results of three typical one dimensional axial flow velocity measurements are presented in section 6.1.2 in order to help illustrate the volume flow measurement method. The method used to determine a set of constant axial velocity points from the one dimensional axial flow velocity measurements is described in section 6.1.3.

6.1.1 Typical Fully Developed Laminar Flow

A perspective view of the two dimensional flow velocity field in a typical round tube with constant laminar flow is shown in Figure 18. The figure is the result of a simulation of a typical flow profile which will be measured using the time domain technique described in Chapter 5, and the flow regulation system described in Chapter 4. The flow velocity field for a unit radius tube is given by

$$V(r) = V_0(1 - r^2) \quad (59)$$

where r is the distance from the tube center and V_0 is the peak velocity at the center of a unit radius tube. Figure 18 is a two dimensional perspective representation of this flow velocity field. Along any line passing through the tube the flow velocity

profile will have a parabolic shape [39] as illustrated by Figure 18. Thus, the one dimensional flow measurement method should provide flow profiles which are parabolic in shape as long as fully developed laminar flow exists.

The volume flow measurement problem is to estimate the velocity field as accurately as possible from the one dimensional flow profiles and then estimate the volume flow. Since the flow velocity is zero at the tube walls and in the region outside the tube, the two dimensional velocity field is spatially limited. If this velocity field is considered a two dimensional signal, then this continuous signal has an infinite bandwidth since it is spatially limited [45]. Because of the infinite spatial bandwidth, the spatial sampling rate used to represent this velocity field must be infinite in order to reconstruct the velocity profile exactly. Thus, any finite spatial sampling rate which can be used in practice will result in an inaccurate estimate of the flow velocity field. The accuracy of the volume flow estimate obtained from samples of the flow velocity field is one of the major objectives of the simulation results presented in section 6.5 and the experimental results to be considered in Chapter 7.

6.1.2 Typical One Dimensional Axial Flow Velocity Measurements

In order to estimate the volume flow in a round tube from one dimensional axial flow velocity measurements, it is important to examine the one dimensional axial velocity results and determine how well they fit the shape of the flow profile

predicted by equation (59). Axial flow velocity measurements similar to those presented in this section can be found in [12] where buttermilk was used as a scattering fluid. More detailed discussion of the one dimensional axial flow velocity results can be found in Chapter 7, section 7.1.

Figure 19 shows three average axial flow velocity versus range measurements for a steady flow of porcine blood through a tube with a diameter of approximately 6.3 mm (21 wavelengths). Three different transducer scan angles (α) are shown, one with the beam axis through the tube center ($\alpha = 0.0^\circ$) and two equally spaced on each side at $\alpha = \pm 2.9^\circ$. In each case the measurement angle (θ) was 60° . These measurements were obtained using 384 echoes and the echo data were processed with the algorithm outlined in Figure 17. The transducer had a beamwidth of two wavelengths (0.8 mm in water) at the focus and the distance from the rotation axis to the center of the tube was 40 mm.

The two profiles at $\alpha = \pm 2.9^\circ$ (Figures 19a and 19c) show a decreased width and reduced flow compared to the $\alpha = 0.0^\circ$ profile (Figure 19b). This would be expected as the transducer is scanned away from the center of the tube. The shape of each of the three velocity profiles appears to be very close to the parabolic shape expected. In fact, second order least squares curve fitting of the data points reveals that the mean squared error between the best fit parabolic curve and the measured data is less than 2% of the average axial flow velocity in each case.

6.1.3 Constant Axial Velocity Positions

The axial flow velocity estimates versus range along the ultrasound beam are used to determine the range positions where the velocity is a given value. The ranges where the velocity is a given value are then used to generate a set of constant velocity points. As illustrated in Figure 19, the one dimensional flow measurements form parabolic velocity versus range graphs. A number of velocity values or levels are then used to develop a contour map of the flow velocity profile.

Each positive velocity level (less than the maximum) of each one dimensional measurement has two range positions where the axial velocity has the same value (see Figure 19). A spatial set of these constant velocity points is developed by converting the axial velocity versus range measurements (similar to Figure 19) to the ranges at which the axial velocity is equal to a series of velocity levels. The polar coordinates (range of a velocity level and scan angle) are then converted to rectangular coordinate values by using the known distance from the scan axis (defined in Chapter 4) to the transducer. When the positions of constant velocity values for all angular positions are plotted on a rectangular grid, a result similar to that shown with Xs in Figure 20 is obtained. Each of these Xs on the same ellipse represents the spatial position of the same velocity value.

The constant axial velocity points can now be obtained from a set of one dimensional axial flow velocity measurements. All of the constant velocity points lie in a plane defined as the

measurement plane. The measurement plane is the plane which contains each of the scan lines as the ultrasonic beam moves across the vessel and is the plane shown in Figure 20. The measurement angle (θ_M) is the angle between the measurement plane and the flow velocity vector. The measurement plane is perpendicular to a plane which passes through the scan axis and the center of the vessel. The measurement plane is, therefore, perpendicular to the plane shown in the cross sectional view of the scan axis and vessel shown in Figure 11. The velocity versus range data are collected at a number of different scan angles. The velocity versus range data are then converted to range versus velocity by first finding the velocity points closest to the velocity level. Since 25 velocity versus range values are measured every 0.6 mm, four of the range values are close to a particular velocity level. Linear interpolation is then used to estimate the range location of the velocity level. The X axis in the measurement plane is defined as the beam path when the scan angle is zero. The angle, PHI, is defined as the angle in the measurement plane between the ultrasonic beam and the X axis. The angle in the measurement plane, PHI, is related to the scan angle, α , by the sine of the track setting or $(R_0 + F)/(R_Z + F)$. Figure 11 shows these three distances for the circular track. Note that R_0 is negative because the scan axis is in front of the transducer. Given the polar coordinate values (R, α) for a particular velocity level, the Cartesian coordinate values (X, Y) in the measurement plane are determined as follows:

$$\text{PHI} = \frac{R_0 + F}{R_z + F} \propto$$

$$X = (R + R_z) \text{ Sin}(\text{PHI})$$

$$Y = (R + R_z) \text{ Cos}(\text{PHI}) \quad (60)$$

where, as shown in Figure 11,

R_0 = distance from scan axis to front face of transducer at $\theta = 90^\circ$,

R_z = distance from scan axis to front face of transducer along a path colinear with the axis of the beam, and

F = distance from front face of transducer to center of rotation of the circular track.

The value of R_z changes depending on the position of the transducer on the circular track. The constant R_0 is determined by the machining of the circular track and the mounting of the transducer. Grooves to indicate the track setting, θ , were etched accurately on the circular track at 90° , 75° , 60° , and 45° . The accuracy of these grooves was verified by arc length and diameter measurements of the track using calipers with an accuracy of ± 0.1 mm. The track setting grooves were found to be accurate to within $\pm 1^\circ$. For the circular track used in this study, caliper measurements revealed that $F = 49.4$ mm and $R_0 = -9.5$ mm and both remained constant throughout the experiments. Measurements of the circular track with an accuracy of ± 0.1 mm have revealed the values for R_z versus track setting shown in Table 5.

Table 5 - R_z Measured Values Versus Track Setting.

Track Setting (θ)	R_z (mm)
90°	-9.5
75°	-8.1
60°	-3.3
45°	7.0

Figure 20 is an illustration of the constant velocity ellipses of steady laminar flow which exist in a round tube for a measurement angle of 45°. Seventeen different scan angles are shown with five different constant flow contours. The outer most curve indicates where the tube wall is and each concentric ellipse towards the center indicates a greater velocity. For this example, starting with the outer most ellipse and moving inward, the velocity levels represented by each ellipse are 0, 0.1 V_0 , 0.3 V_0 , 0.5 V_0 , and 0.7 V_0 . The area inside the center ellipse represents where the velocity is greater than 70% of the maximum value.

The simulated constant velocity ellipses shown in Figure 20 are what would be expected from an ideal round tube scanned by an ideal ultrasonic measurement system at a 45° measurement angle. In an actual experiment, data similar to the data presented in Figure 19 were used to determine the volume flow. The volume flow estimate must be determined completely from the constant velocity points obtained from the ultrasonic measurements. The main assumption made in the development of the volume flow estimates used in this research is that the vessel has a circular

cross section. Since the vessel is assumed to be circular and the axial velocity measurements are made at an oblique angle, the constant velocity contours form a set of constant velocity ellipses as shown in Figure 20. Thus, in order to estimate the volume flow, the vessel size, measurement angle, and the axial flow velocity field must all be estimated from the ultrasonic measurements. The elliptic fitting algorithm used to estimate the size and orientation of the constant velocity ellipses from the constant velocity points is described in the next section. The methods used to determine the volume flow from the size of the constant velocity ellipses are considered in section 6.3.

6.2 ELLIPTIC FITTING ALGORITHM

This section contains the details of the algorithm used to obtain the parameters of the constant velocity ellipses (size, position, and orientation) from the ultrasonic axial velocity flow measurements. For a tube with a perfectly circular cross section and constant laminar flow, the constant velocity curves form a set of concentric ellipses when the measurement plane is at an oblique angle. The data points measured by ultrasound represent only a portion of each ellipse. Within the section of the ellipses near the tube wall which are nearly parallel to the beam, no ultrasonic measurement of the velocity can be made because most of the ultrasonic energy is reflected away from the vessel by the tube wall.

The minor axis of each constant velocity ellipse represents the diameter of the region in the tube where the flow velocity is greater than the particular velocity level. The minor axis of the ellipse at a zero velocity level is the diameter of the tube. Since the minor axes of all of the constant velocity ellipses are nearly perpendicular to the ultrasonic beam (see Figure 20), it is not possible to directly determine the dimensions of the tube or the constant velocity ellipses. Other scanning geometries, such as a transducer which moves around the center of the tube or a transducer which moves along the circular track, could be used to directly measure the tube dimensions. Since these scanning methods are much more complicated and require additional motion of the transducer, such scanning methods are not practical for blood flow measurement in the human body and will not be considered here. It is therefore important to estimate the dimensions of the constant velocity ellipses from a limited number of scan angles in the geometry illustrated in Figure 20.

One way to estimate the dimensions of the constant velocity ellipses is to fit an elliptical curve to the data points. Although this is somewhat difficult to accomplish for the general case of arbitrarily distributed data points, many simulations have shown that a typical ellipse can be fitted by least squares curve fitting.

Figure 21 shows a flow chart of the elliptic fitting procedure. The equation of an ellipse which is rotated by ANG with a center at (X_0, Y_0) and axis lengths $2a$ and $2b$ is as follows:

$$A(X - X_0)^2 + 2B(X - X_0)(Y - Y_0) + C(Y - Y_0)^2 = 1 \quad (61)$$

where

$$A = \frac{\cos^2(\text{ANG})}{a^2} + \frac{\sin^2(\text{ANG})}{b^2}$$

$$B = \sin(\text{ANG}) \cos(\text{ANG}) (b^{-2} - a^{-2})$$

$$C = \frac{\cos^2(\text{ANG})}{b^2} + \frac{\sin^2(\text{ANG})}{a^2}$$

A typical example of an ellipse described by the above equation is shown in Figure 22 in which the five parameters (a , b , X_0 , Y_0 , and ANG) are graphically defined.

The elliptic fitting problem is to estimate A , B , C , X_0 and Y_0 from the experimental (x_n, y_n) data. The rotation angle, ANG , is the most difficult parameter to estimate, since for a nonzero rotation angle ($\text{ANG} \neq 0$), the cross product term B is nonzero. Thus, a simple way to estimate the rotation angle is to rotate the data by $-\text{ANG}$ and determine how well the data fit an unrotated ellipse ($B=0$). By trying a series of ANG values, the rotation angle can be estimated. The procedure illustrated in Figure 21 evaluates 21, one degree increments from -10° to $+10^\circ$. A measure of the difference between the N data points $(x_n, y_n; 0 \leq n \leq N-1)$ and the best fit unrotated ellipse is the mean square error (MSE) defined as follows:

$$\text{MSE} = \frac{1}{N} \sum_{n=0}^{N-1} (y_n - Y_0 \pm b \sqrt{1 - a^{-2}(x_n - X_0)^2})^2 \quad (62)$$

where the sign in equation (62) is determined from (1) the geometry and (2) the sign of $(x_n - X_0)$. The value of ANG which

has the lowest MSE is used as the final estimate of the rotation angle.

The other four parameters (X_0, Y_0, a, b) are estimated by the iterative technique illustrated in Figure 21. The first center estimate is determined from the mean of the x_n and y_n data points,

$$(X_0, Y_0) = \left(\frac{1}{N} \sum_{n=0}^{N-1} x_n, \frac{1}{N} \sum_{n=0}^{N-1} y_n \right) \quad (63)$$

This initial center estimate requires that the data points are reasonably evenly distributed on both sides of the ellipse. If only one side of the ellipse were represented by the data, then a large error in this initial guess of the center would result. Since the center estimate is subtracted from the data before least squares curve fitting, the iterative technique may not converge to the proper values if the initial center estimate is not close to the actual center. Extensive simulation of the elliptic fitting procedure has revealed that as long as some of the data points represent both sides of the ellipse and at least five data points are used, the procedure converges to the correct center solution (within 1% of the actual value). If fewer than five data points are used, the parameters determined by the procedure are not unique (five parameters are being determined) and the values obtained from the algorithm are then unpredictable. A simulation of the overall accuracy of the volume flow method including the elliptic fitting procedure will be discussed in section 6.5.

The next step in the procedure is to assume that one of the center coordinates is zero and fit the square of the data points versus the other coordinate by second order least squares. For example, if $X_0=0$, the data (x_n^2, y_n) are a parabolic curve. Figure 23 illustrates this case. The unrotated ellipse shown in Figure 23 has $(a, b, X_0, Y_0) = (1.5, 2.0, 0.0, 0.1)$. The curve (x^2, y) is a parabola and (y^2, x) is composed of two parabolic curves which are superimposed. The fit of (x^2, y) yields estimates of A_0, A_1 , and A_2 of $x^2 = A_0 + A_1 y + A_2 y^2$. The A_0, A_1, A_2 values can be used to determine Y_0, a , and b as shown in the flow chart. This Y_0 value is subtracted from the data and a fit of (y^2, x) is then used to estimate X_0, a and b . The X_0 value is then subtracted from the x_n data points. The least squares fit and center subtraction are repeated ten times for each angle. This number of iterations was found to be sufficient for convergence of the a, b, X_0 , and Y_0 values to within 1% of the correct values in all of the simulations. As shown in Figure 21, after each least squares estimate of a and b , the MSE is again estimated according to equation (62). If the current MSE is lower than all previous MSE values (MSEMIN), the current values of a, b, X_0, Y_0 , and ANG replace the final values passed to the calling program. Since 420 (21 different ANG values and 20 curve fits for each angle) least squares second order curve fits must be performed for each elliptic fit, the time to determine the parameters of one 30 point ellipse was 7 seconds on the host computer (VAX 11/730). The five parameters passed to the calling program are the estimate of (a, b, X_0, Y_0, ANG) for $ANG = -10^\circ$ to 10° with the smallest mean squared error.

6.3 VOLUME FLOW ESTIMATES

The constant velocity ellipses obtained with the elliptic fitting algorithm described in the last section can now be used to estimate the volume flow. The set of parameters obtained from each constant velocity ellipse provides a great deal of information about the two dimensional flow velocity field. The estimation of volume flow from the parameters obtained from elliptical curve fitting is described in this section. Estimates of the measurement angle, tube size, and constant velocity ellipse parameters can be used to determine two volume flow estimates, namely the INTEGRATED and TUBE SIZE estimates.

The center, angle of rotation, and major and minor axes of each elliptical estimate of constant velocity contour are determined by the elliptic fitting algorithm. The measurement angle was constant for all of the constant velocity levels so that the ratio of the axes of the ellipses (or the aspect ratio) should be approximately the same. However, because of the variation in the experimental axial velocity measurements, each resulting constant velocity ellipse will not have the same aspect ratio. An average of the measurement angle estimates obtained from each ellipse may provide a good measurement angle estimate. The average measurement angle estimate is given by

$$\theta_M = \frac{1}{M} \sum_{n=0}^{M-1} \sin^{-1} \left[\frac{a_n}{b_n} \right] \quad (64)$$

where a_n is half the minor axis length (see a and b in Figure 22) and b_n is half the major axis length of the n^{th} constant velocity

ellipse. A total of M ellipses are determined with the first and largest ellipse, representing the wall, where the flow velocity is estimated to be zero, having $n=0$. The volume flow can be estimated by numeric integration as follows:

$$Q = \frac{\pi}{3} \sum_{n=1}^{M-1} \frac{(V_n - V_{n-1}) (a_{n-1}^3 - a_n^3)}{(a_{n-1} - a_n) \cos(\theta_M)} + \frac{\pi a_{M-1}^2 (V_{\max} - V_{M-1})}{2 \cos(\theta_M)} \quad (65)$$

where

- Q = total volume flow (m^3/sec),
- a_n = half of the minor axis of ellipse n ,
- b_n = half of the major axis of ellipse n ,
- V_n = axial velocity level of ellipse n (m/s),
- V_{\max} = maximum axial velocity value of all one dimensional measurements (m/s), and
- M = number of constant velocity ellipses.

Equation (65) approximates the volume of each layer as the bottom circular section parallel to the base of a right circular cone as illustrated in Figure 24. Each of the $M-1$ layers is added in the first term of equation (65). The thickness of the layer is determined by the difference in the velocity levels ($V_n - V_{n-1}$) and the minor axes of the ellipses determine the size of the top and bottom surfaces of the circular conic section. The approximation assumes that the flow between constant velocity ellipses increases in a linear fashion. Equation (65) assumes that the velocity values of the ellipses are ordered from the smallest (V_0) to the largest (V_{M-1}) and that the size of each ellipse is smaller than the previous ellipse.

A simulation of the errors associated with equation (65) (to be discussed in section 6.5) has shown that $M=10$ provides an accurate estimate of the flow to within $\pm 2\%$. Given that $M=10$, the first term of equation (65) is the flow due to all of the nine conic slices from the wall to 90% of the peak flow. The flow levels were chosen at multiples of 10% of the peak axial flow velocity (V_{\max}), or $V_n = 0.1 V_{\max}$. The peak flow (V_{\max}) is the maximum axial flow velocity of all of the one dimensional measurements. The second term is the additional flow due to the difference between the maximum flow (V_{\max}) and the highest constant velocity ellipse value (V_{M-1}). The shape of this top section, as illustrated in Figure 24, is assumed to be parabolic and axially symmetric. Since the V_{M-1} ellipse is the smallest ellipse, it represents the smallest contribution of the total flow, Q . The largest contribution is from the first and largest section. Thus, from many one dimensional flow measurements, the total flow may be estimated without prior knowledge of the vessel size, or transducer orientation. When equation (65) is used to estimate the volume flow and $M=10$, the result will be called the INTEGRATED volume flow estimate.

The time required to determine the parameters of the ten ellipses and the total volume by equation (65) is rather long (78 seconds on the host computer). Since computationally simpler volume flow estimates than that obtained using ten ellipses ($M=10$) require less time, it is intriguing to determine the accuracy of other simpler volume flow estimates. Another estimate of the volume flow can be obtained by using only the

last term in equation (65) and $V_{M-1}=0$. Thus, the entire flow profile is assumed to be parabolic and axially symmetric. This simpler estimate is obtained by setting $M=1$ in equations (64) and (65) and will be called the TUBE SIZE volume flow estimate. The measurement angle must still be determined by equation (64) and the tube radius (a_{M-1} or a_0) must be determined by elliptic fitting the tube wall data points. The accuracy of this TUBE SIZE flow estimate and the INTEGRATED flow estimate determined from a ten layer integration ($M=10$ in equation (65)) will be compared in the next chapter.

6.4 ERRORS DUE TO REFRACTION

When the speed of sound in the flowing fluid is different than the speed of sound in the surrounding medium, the ultrasonic beam bends due to refraction. Although this effect should not be significant for Sephadex^R flow experiments (because the speeds are nearly the same, see section 4.6.2), the speed of sound in porcine blood (1.55 mm/ μ s) was different from the speed of sound in the surrounding medium (normal saline was used in blood flow experiments with a speed of sound of 1.51 mm/ μ s). The methods used to address the problems caused by the refraction of the ultrasonic beam are described in this section. A simulation will be used to show the change in the beam direction due to refraction in section 6.4.1. Also, a method which can correct the experimentally measured data for the effects of refraction will be considered in section 6.4.2.

6.4.1 Refraction Simulation

Figure 25 shows a schematic representation of an ultrasound beam (in terms of ray theory) incident at an oblique angle, θ_I , on the round tube. If the beam size is small compared to the size of the tube, Snell's law can be used to predict the path of the refracted beam. Since Cartesian coordinates are used throughout this discussion, the formulation of the direction of the beam is easier to express in terms of the slopes (dY/dX) of the beam and tube wall instead of angles between them. The angle between the incident beam (slope $M1$) and the tube wall (slope $M2$) is θ_I . The angle between the tube wall (slope $M2$) and the refracted ray (slope $M3$) is θ_M . By using the dot product of the vectors, the following relations between the angles and the slopes can be derived:

$$\cos(\theta_I) = \frac{1 + M1 M2}{\sqrt{(1 + M1^2)(1 + M2^2)}} \quad (66)$$

$$\cos(\theta_M) = \frac{1 + M3 M2}{\sqrt{(1 + M3^2)(1 + M2^2)}} \quad (67)$$

By Snell's law,

$$\frac{\cos(\theta_M)}{\cos(\theta_I)} = \frac{c_1}{c_0} \quad (68)$$

where c_1 is the speed of sound in the medium inside the tube and c_0 is the speed of sound in the surrounding medium. By using

equations (66), (67) and (68), a relationship can be derived between M_3 and M_1 in terms of M_2 . The result is,

$$M_3 = \frac{-M_2 \pm \sqrt{C(M_2^2 - C + 1)}}{M_2^2 - C} \quad (69)$$

where

$$C = \frac{c_1^2 (1 + M_1 M_2)^2}{c_0^2 (1 + M_1^2)}$$

The sign in equation (69) must be determined from the geometry of the system. For the case illustrated in Figure 25, the positive sign is correct.

Figure 26 shows simulation results for cases with the transducer perpendicular to the flow direction for c_1/c_0 equal to 0.90 and 1.10. For an index of refraction greater than one, the rays bend away from the tube (Figure 26a). An index of refraction less than one causes the rays to bend inward toward the tube center (Figure 26b). In both cases, a total of 17 scan angles were simulated and the transducer was at (-12, 0) or 12 units away from the center of the unit radius tube. This relative transducer location is typical for the experimental measurement to be considered in the next chapter. The dashed lines beyond the back wall of the tube indicate the path of the ultrasound beam in the tube. Since the path of the ultrasound beam beyond the back wall does not produce a backscattered signal, the refraction at the back wall was not simulated. Figure 26 illustrates the rather large change in the direction of the ultrasonic beam when only a 10% change in the speed of sound occurs.

6.4.2 Refraction Correction

Axial flow velocity measurements of fluids with a different speed of sound than the surrounding fluid have the same parabolic flow profile as measurements of fluids with no refraction effects. The path of the refracted ultrasonic beam inside the tube is straight but in a different direction than the incident beam. Thus, the width of the flow profile and the magnitude of the velocities are different than what would be expected if the bending of the ultrasonic beam were not included in the analysis of the experimental data. In order to obtain accurate flow estimates for experiments where refraction effects are important, some method to correct the experimental data for the refraction effects is required.

Since the incident angle (θ_I) between the ultrasonic beam and the tube wall is unknown for a particular scan angle, it must be determined from the ultrasonic data. By determining the front wall locations from the experimental data, an initial estimate of the incident angle in the measurement plane can be obtained. An iterative method can then be used to estimate the incident angle for each scan angle. Then, by using the measured values of the speed of sound in the materials used in a particular experiment, the ultrasound beam direction inside the vessel can be determined. The following steps were used to correct the experimental axial flow velocity measurements for the effects of refraction:

1. Locate the front wall from one dimensional velocity estimates assuming constant speed of sound (c_0) for entire range. The wall location is extrapolated from a second order least squares fit of the one dimensional velocity values, as was described in section 6.1. The location of the front wall should not be affected by a different speed of sound inside the tube.
2. Correct the one dimensional estimate by using c_1 for the speed of sound for range cell locations after the front wall position determined above.
3. Determine the initial elliptic fit of front and back wall data using the corrected one dimensional results. This wall estimate is used as an initial guess of the front wall where the refraction of the beam occurs.
4. Using the slope of the front wall of the wall ellipse, correct the path of the ultrasonic beam using equation (69) and determine the location of the refraction corrected back wall.
5. Using the new back wall estimates, determine the best fit ellipse for the tube wall. These tube wall values replace the old wall ellipse values.
6. Repeat steps 4 and 5 until the size of the wall ellipse does not change significantly (less than 1% change in the a_0 value). For refraction indices as large as 1.15, eight iterations of steps 4 and 5 were sufficient. Additional